Market and welfare effects of quality misperception in food labels

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Abstract

Information-based policies, most prominently labels, reveal credence attributes of food products and, presumably, help consumers make better choices by reducing their misperception of product quality. However, much remains unexamined regarding how firms' strategic reactions to consumers' misperception of quality influence the benefits of information-based policies. We consider an oligopoly model where heterogeneous consumers can over- or under-estimate the quality of products in the market, and firms choose quality and prices conditional on consumers' perception of quality. We find that, under empirically prevalent conditions, misperception can increase efficiency in relation to the perfect information case; it does so if 1) it strengthens firms' incentives to provide higher quality, countervailing the chronic underprovision of quality that prevails under perfect information or 2) it galvanizes competition, reversing another deleterious effect of product differentiation, namely high quality-adjusted markups that restrain commerce. Our results imply that information-based policies aimed at curbing misperception, such as requiring or allowing (under voluntary certification) additional information, nudging, and changes in label format, can have deleterious effects on efficiency and, perhaps more importantly, hurt the very consumers they mean to protect.

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Empirical evidence indicates that consumers routinely misperceive the quality of food products. As a result, they overvalue (overestimate quality) some food products (e.g., Lee et al., 2013; McFadden and Lusk, 2018; Bernard et al., 2019) and undervalue (underestimate quality) others (e.g. Kiesel and Villas-Boas, 2013; Rainie et al., 2015; Liaukonyte et al., 2015) relative to their competitors along the quality spectrum. Conventional wisdom dictates that misperception distorts consumers' choices and reduces their welfare. Regulators and the public have favored information-based policies, more prominently labels, to inform consumers and help them make better choices (Roe and Sheldon, 2007; USDA, 2013; Bonroy and Constantatos, 2015; NYTimes, 2017; Lusk, 2018). Yet, much remains unexamined regarding firms' strategic responses to misperception and, consequently, the effectiveness of information-based policies to protect consumers and enhance overall market efficiency.

We consider a market in which two single-product firms compete on quality and prices. Following Bonroy and Constantatos (2015), we assume quality is determined by credence attributes and these attributes are conveyed by labels, but consumers may misperceive the information in those labels. We show that producers' strategic reactions to consumer misperception can increase welfare of some consumers and even raise market efficiency relative to a situation without misperception. This may seem surprising at first glance. After all, misperception leads consumers to make incorrect choices (Villas-Boas et al., 2020), and it leads firms to extract informational rents from misinformed consumers. However, we find that firms' strategic reactions to misperception lead to higher efficiency if they: 1) raise the average quality offered in the market partly correcting the underprovision of quality that prevails under imperfect competition in the absence of misperception; or 2) expand the size of the market enough to offset reductions in average quality.

Strategic reactions raise the average quality offered in the market when misperception provides incentives for firms to offer higher qualities. In our duopoly model with single-product firms, this takes place when consumers overvalue either labeled product. When consumers overvalue the higher-quality product, the firm offering this product raises its quality (because returns to quality are higher and the firm can also increase markup without a significant loss in market share), to which the follower responds by raising its own quality (because it allows it to increase markup). In other words, the increase in quality of the high-quality product "pulls" the low-quality product up the quality spectrum. Governed by the same forces, when consumers overvalue the low-quality product, the firm offering this product raises its quality and "pushes" the higher-quality product up the quality spectrum in equilibrium.

We also find that misperception can still result in higher efficiency, even if it does not induce a rise in the average quality offered in the market, as long as firms' strategic reactions to misperception lead to an expansion in the market for labeled products. In our duopoly model with single-product firms, this can happen when consumers undervalue the high-quality labeled product and simultaneously overvalue the low-quality labeled product; in other words, consumers under-estimate product differentiation. As a result, misperception *can* (under conditions we formally identify) push products sufficiently close to each other on the quality spectrum, so that price competition delivers an expansion in the market for labeled products in equilibrium. In this situation, while misperception does not help reverse the underprovision of quality that prevails in its absence, it does galvanize competition intensity which reverses another deleterious effect of product differentiation, namely high quality-adjusted markups that restrain commerce.

A key corollary of our findings is that, under empirically prevalent conditions, information-based policies that seek to curb misperception (e.g. stricter labeling rules, allowing uncertified private labels, changes in the format information is presented, nudging, etc.) may reduce efficiency. But, perhaps more importantly, these interventions may also harm the very consumers they mean to buttress. We find that interventions that reduce misperception invariably hurt at least one consumer segment. In fact, in many cases, the losses in this consumer segment exceed gains in other segments, reducing total consumer surplus. Reduction of misperception harms two consumer segments: consumers that purchase a product that is undervalued before the intervention; or consumers that purchase a product that competes with another that is overvalued before the intervention.

There are many examples of information-based policies that seek to curb misperception and that, based on our findings, seem likely to be detrimental for consumer surplus and efficiency. The Organic Foods Production Act of 1990 and the subsequent rules for organic certification are likely to: decrease the average quality offered in the market due to undervaluation of the organic label in reference to the "made with organic" label (Streletskaya et al., 2019); or weaken price competition due to reduced valuation of the organic label in conjunction with increased valuation of the 100% organic label. FDA's 2003 requirement for labeling the presence of Trans-Fat reduces consumers' valuation of these products relative to other alternatives along the quality spectrum (Kiesel and Villas-Boas, 2013), which may erode incentives to reduce Trans-Fat when they are not fully eliminated.

Other prominent examples include California's Proposition 37 of 2012 and Vermont's Act 120 of 2016, as well as the National Bioengineered Food Disclosure Standard of 2016 requiring firms to disclose when products contain GMOs. This is likely to cause undervaluation of these products relative to others without the "contain" label (Liaukonyte et al., 2015; Villas-Boas et al., 2020). Finally, examples are not only constrained to enacted policies but also include proposed policies. To simplify information in labels, many have advocated for visual cues that replace rather complicated information; for instance, adding a green-, yellow-, and red-colored stickers to products to reflect their nutritional quality. Strategies that simplify information can result in consumers overestimating the difference in quality across products (Villas-Boas et al., 2020).

Our study contributes to a rather thin literature on market and welfare effects of quality misperception. Studies in this literature model very specific situations both in terms of the nature of misperception, as well as the nature of quality competition. Regarding the nature of misperception, previous studies model misperception as either consumers' inability to distinguish between quality grades (e.g. Brécard, 2014; Buehler and Schuett, 2014; Brécard, 2017) or, specifically motivated by eco-labeling, overvaluation of medium quality products (e.g. Baksi et al., 2017). Contrarily, empirical evidence suggests that misperception of quality in food products can manifest as under- or over-valuation of goods located anywhere in the quality spectrum. We develop a framework that considers these types of misperception in a systematic way and show that different types of misperception can have drastically different market and welfare implications.

Regarding the nature of competition, the strategic environment in which firms operate can vary widely across food markets. In some markets, firms have the ability to credibly commit to a choice of quality and reveal it to its competitors, effectively turning the quality-competition stage into a sequential game (Aoki and Kurz, 2003). A recent example of this is the move by some poultry firms to build capacity to produce cage-free eggs (e.g. EggIndustry, 2019) in order to serve retailers and restaurants that took a pledge to demand only cage-free eggs by 2025 (Lusk, 2019). In other cases, firms do not have the ability to credibly commit to a choice of quality before others and the quality-competition stage resembles a simultaneous game. An example of this is competition on nutritional content (e.g. non-fat yogurt; no trans-fat) where biochemical processes underlying qualities are well-understood by all firms. A key feature underlying the ability of firms to commit to a quality level is the cost of switching between these levels, influenced by sunk costs and asset specificity; if cost of switching is high (low) firms can (cannot) credibly commit to a choice of quality.

Despite the empirical ubiquity of sequential and simultaneous quality competition and the fact that the nature of quality competition is a crucial factor governing equilibrium in markets without misperception (e.g. Aoki and Prusa, 1997, Lehmann Grube, 1997), previous studies that incorporate consumer misperception only considered simultaneous quality choice. We differentiate from those studies and examine both simultaneous and sequential quality competition and study their interaction with the nature of misperception.

In sum, we find that quality misperception translates into changes in demand which, in turn, trigger strategic responses by firms. These strategic responses change qualities and prices offered in equilibrium. Consequently, information-based policies seeking to curb consumer misperception shift qualities and prices in equilibrium. We formally characterize these changes in equilibrium and find that the effects of common information-based policies on consumer surplus (total and by consumer segment) and efficiency depend crucially on the nature of misperception as well as the nature of competition. Policies that reduce misperception are likely to be harmful if consumers overestimate the quality of a product, or if the intervention raises (reduces) perceived product differentiation when quality competition is simultaneous (sequential). Thus, our analysis suggests that information-based policies should contemplate not only demand-side forces like the type of misperception likely to prevail in the market, but also supply-side ones like sunk costs and asset specificity associated with production of higher quality levels.

The paper is structured as follows. Section 1 formally introduces the models and our equilibrium concepts. Sections 2 and 3 describe the market and welfare effects of misperception, respectively. Section 4 discusses the implications for information-based policies in the United States and Section 5 concludes.

1 Model

1.1 The Demand Side: Heterogenous Consumers and Misperception

Consider a market where consumers differ in their taste for quality, denoted by v, and are distributed along a continuum of unit length depicting willingness-to-pay (WTP) for quality, θ. Consumers are distributed along the continuum according to a uniform probability distribution function with unit density. Two single-product firms operate in the market, so consumers can choose between two labeled products and an outside good, and they buy a single unit of the good they choose to consume.

Quality attributes are credence (e.g. effect of consumption on health, the environment, animal and human welfare, etc.) and, hence, unobservable to consumers. However, the quality grade v of each labeled product is certified by a non-profit, credible third-party. This third-party uses a continuous grade program to certify quality. Misperception can arise from imperfect disclosure of information or imperfect understanding of information included in labels (Brécard, 2017). We follow many previous studies (Ben Youssef and Abderrazak, 2009; Harbaugh et al., 2011; Brécard, 2014, 2017) and assume certifiers are honest and do not act strategically. Also, we do not endogeneize the decision of what to label (e.g. Forlin, 2020) and instead focus on misperception arising from consumers' inability to fully understand information in labels; a phenomenon widely documented in the literature (see a discussion in Zilberman et al., 2018).

Misperception creates a misalignment between perceived and actual product quality. Because we only have two labeled products in the market, the quality of the product certified with the relatively higher quality grade is represented by v_h, and a relatively lower quality grade is represented by v_l; i.e. $v_h > v_l$. We also formally introduce two misperception parameters, k_h and k_l. We interpret these as wedges between perceived and actual quality; perceived qualities are denoted by $k_h v_h$ and $k_l v_l$ for the high- and low-quality products, respectively. In the absence of misperception regarding quality of product j, we have $k_j = 1$; in the presence of overvaluation $k_j > 1$; and in the presence of undervaluation $k_j < 1$.

As suggested by empirical evidence, misperception can increase or decrease the perceived difference in quality between products, and it can also increase or decrease the perceived average quality of products offered in the market. These are important distinctions because market efficiency is related to the average quality offered in the market but also the intensity of price competition, which is influenced by the perceived difference in quality. Different types of misperception can have disparate effects on these forces, making a systematic analysis of misperception sources crucial.

With these considerations in mind, we study six types of misperception. First, we consider a case in which consumers overvalue the high-quality product only, i.e. $k_h > 1$ and $k_l = 1$; this raises the perceived difference in quality between products while also increasing the perceived average quality of products in the market. Second, we consider a case in which consumers overvalue the low-quality product only, i.e. $k_h = 1$ and $k_l > 1$; this reduces the perceived difference in quality between products while also increasing the perceived average quality of products in the market. Third, we consider a case in which consumers undervalue the high-quality product only, i.e. $k_h < 1$ and $k_l = 1$; this reduces the perceived difference in quality between products while also reducing the perceived average quality of products in the market. Fourth, we consider a case in which consumers undervalue the low-quality product only, i.e. $k_h = 1$ and $k_l < 1$; this raises the perceived difference in quality between products while also reducing the perceived average quality of products in the market. Fifth, we consider a case in which consumers overvalue the high-quality product and undervalue the low quality in the same magnitude; this raises the perceived difference in quality between products but without affecting the perceived average quality of products in the market. Finally, we consider a case in which consumers undervalue the high-quality product and overvalue the low quality in the same magnitude; this reduces the perceived difference in quality between products but without affecting the perceived average quality of products in the market.

Armed with these representations of misperception, we modify the class of indirect utility functions (Bonroy and Constantatos, 2015) presented initially by Jean Tirole (1988) and subsequently pursued by Ronnen (1991) and Lehmann Grube (1997) by altering their definition of consumer's utility. We let the indirect utility of consumers that buy labeled quality grade j be $V_i(v_j, p_j, k_j) = \theta_i k_j v_j - p_j$, where i is an index of the consumer's position in the WTP distribution, θ_i is the consumer's valuation of quality, $k_j v_j$ is consumers' perceived quality of product j ($j = l$ for the low-quality labeled product and $j = h$ for the high-quality labeled product), and p_j is the price paid for product j. For the treatment in which misperception is on low-quality grades, $k_h = 1$. For treatments in which misperception is on the high-quality grade, $k_l = 1$. We normalize the indirect utility of those consumers consuming the outside good to zero.

The consumer that is indifferent between buying the low-quality product and the outside good, given by θ_{0l}, can be found by setting the indirect utility of these options equal, such that $\theta_{0l} k_l v_l - p_l = 0$. This implies $\theta_{0l} = \frac{p_l}{k_l v_l}$. By the same procedure, the consumer that is indifferent between buying the low-quality and high-quality is $\theta_{lh} = \frac{p_h - p_l}{k_h v_h - k_l v_l}$. These expressions determine the market for low- and high-quality labeled products. Aggregate demands are given by $D_h(v_h, v_l, p_h, p_l; k_h, k_l) = \int_{\theta_{lh}}^{1} d\theta = 1 - \theta_{lh}$, and $D_l(v_h, v_l, p_h, p_l; k_h, k_l) = \int_{\theta_{0l}}^{\theta_{lh}} d\theta = \theta_{lh} - \theta_{0l}$ for high- and low-quality products, respectively. It is clear from these expressions that the quality of labeled goods and their price affect their consumption. Both of these emerge endogenously in equilibrium from the interaction between demand and strategic supply responses by firms in this market. We now turn to the supply side of the model.

1.2 The Supply Side: Quality and Price Competition

On the supply side, we consider a market with single-product suppliers. Firms have access to the same technology, which consists of constant marginal cost, normalized to zero. For simplicity let us assume that two firms are active in the market and that they offer products containing one or more credence attributes that are certified through labels. The cost of certification $C(\cdot)$ is independent of the number of units produced but increasing in the quality level. Studies using duopoly models of vertically differentiated products typically assume $C(\cdot)$ is convex and twice continuously differentiable. This assumption guarantees fulfillment of the second order conditions for a maximum and existence of a unique equilibrium in pure strategies (Lehmann Grube, 1997). In our study, consideration of a range of misperception types places an additional burden on tractability. We follow a common practice in the literature (e.g. Motta, 1993; Aoki and Prusa, 1997; Buehler and Schuett, 2014 ; Baksi et al.,

2017) and assume a quadratic cost structure of the form $C\left(v_j\right) = \frac{v_j^2}{2}$ $(j \in h,\ l)$, which renders our model tractable (i.e., capable of generating unambiguously signed comparative statics) across misperception types.[1]

Conditional on the aggregate demand for each product characterized in the previous sub-section, competition between duopolists proceeds in two stages. First, firms choose quality (quality-competition stage) and then, conditional on quality, they compete in prices (price-competition stage). The solution of the two-stage game is characterized by the Sub-Game Perfect Nash Equilibrium (SPNE), which is computed by backward induction; i.e. we first solve for equilibrium prices of the price-competition stage conditional on qualities (equilibrium best-response prices), and then solve for equilibrium qualities conditional on equilibrium best-response prices.

Profits of duopolist firms in the price-competition stage are:

$$\pi_h = R_h\left(v_h, v_l, p_h, p_l; k_h, k_l\right) - C\left(v_h\right) = p_h D_h\left(v_h, v_l, p_h, p_l; k_h, k_l\right) - C\left(v_h\right) \tag{1}$$

$$\pi_l = R_l(v_h, v_l, p_h, p_l; k_h, k_l) - C(v_l) = p_l D_h\left(v_h, v_l, p_h, p_l; k_h, k_l\right) - C(v_l), \tag{2}$$

where $R_j(v_h, v_l, p_h, p_l; k_h,\ k_l)$ stands for revenue of the firm offering product j and the rest is as defined before.

The price-competition stage is assumed to be simultaneous because no firm has a credible mechanism to commit to a specific price before the other firm. Therefore, conditional on quality choices, the solution of the price-competition stage is defined by the Nash Equilibrium of the duopoly Bertrand-pricing game, which consists of a system of two first order conditions: $D_h\left(v_h, v_l, p_h, p_l; k_h, k_l\right) + p_h \frac{\partial D_h(v_h, v_l, p_h, p_l; k_h, k_l)}{\partial p_h} = 0$ and $D_l\left(v_h, v_l, p_h, p_l; k_h, k_l\right) + p_l \frac{\partial D_l(v_h, v_l, p_h, p_l; k_h, k_l)}{\partial p_l} = 0$. With a zero marginal cost of production, the firm raises the price to balance the marginal benefit of a higher markup with the marginal cost of earning that

[1]Our results generalize to other cost structures that are convex, but not of the quadratic form. Numerical simulations that demonstrate this are available from the authors upon request.

markup on a smaller number of units (due to decreased demand). The solution to this system of equations characterizes optimal prices as functions of qualities (the equilibrium best-response prices):

$$p_h{}^* \left(v_h, v_l; k_h, k_l\right) \tag{3}$$

$$p_l{}^* \left(v_h, v_l; k_h, k_l\right) \tag{4}$$

The quality-competition stage is more complex than the price-competition stage. This is because different products are fundamentally different regarding the ability of firms that produce them to switch grades along the quality spectrum. When the cost of switching between different grades is small, firms do not have a credible mechanism to commit to a specific quality before the other firm, given rise to a simultaneous quality-competition game. By contrast, when the cost of switching grades is large, firms can credibly commit to a specific quality giving raise to a sequential quality-competition game. In both cases firms choose quality to maximize profits (1) and (2) subject to equilibrium pricing strategies (3) and (4):

$$\max_{v_h} \pi_h = p_h{}^* D_h \left(v_h, v_l, p_h{}^*, p_l{}^*; k_h, k_l\right) - C(v_h) \tag{5}$$

$$\max_{v_l} \pi_l = p_l{}^* D_l \left(v_h, v_l, p_h{}^*, p_l{}^*; k_h, k_l\right) - C(v_l) \tag{6}$$

Notice that programs (5) and (6) are the same as (1) and (2) but with (3) and (4) inserted in them. The solution to problems (5) and (6) varies according to the nature of the game. More fundamentally, the conditions under which there is a solution (a unique equilibrium in pure strategies) also vary according the nature of the game. We now turn to a description of the procedure by which an equilibrium in quality competition is obtained, and the conditions that guarantee such equilibria exist and are unique.

We start by assuming that firms compete sequentially in the quality game. This assumption is in line with markets in which switching across quality grades is very costly and, thus, a firm can commit to a certain quality level preempting the other from choosing that quality. Which firm chooses first is inconsequential in this case because firms are otherwise homogeneous. In this case, the leader (first mover) chooses a quality and then the follower responds by choosing its own. All of these choices are conditional, of course, on optimal pricing strategies (3) and (4).

The solution to (5) and (6) is obtained by backward induction. First, the follower chooses quality with knowledge of the quality chosen by the leader. Then the leader chooses its quality with knowledge of the follower's best response to its own quality. The leader may choose a high or a low quality. We restrict ourselves to solutions of the quality game in which there is a unique and stable equilibrium in pure strategies. This condition restricts the domain of misperception and it has come to be known as the "no leapfrogging condition" (Lehmann Grube, 1997). It turns out that under the no leapfrogging condition, the leader chooses a high-quality product and obtains higher profits than the follower (Motta, 1993; Aoki and Prusa, 1997; Lehmann Grube, 1997). In the context of our model, we show in Appendix 1 that the no leapfrogging condition holds under the following assumptions:

ASSUMPTION 1. $0.75 < k_h < 1.75$ if $k_l = 1$,

ASSUMPTION 2. $0.58 < k_l < 1.33$ if $k_h = 1$,

Intuitively, assumptions 1 and 2 rule out cases in which misperception is large enough to break the equilibrium that has the leader as the high-quality firm. Conditional on these assumptions, we start by solving problem (6) which yields the follower's best response function $v_l\left(v_h; k_h, k_l\right)$. Subsequently, we solve problem (5) subject to $v_l\left(v_h; k_h, k_l\right)$, which yields the leader's choice of quality in the SPNE of the sequential game,

$$ {v_h}^s\left(k_h, k_l\right) \tag{7} $$

Finally, we insert ${v_h}^s\left(k_h, k_l\right)$ into the best response $v_l\left(v_h; k_h, k_l\right)$ to obtain the follower's choice of quality in the SPNE of the sequential game,

$$v_l{}^s\,(k_h, k_l) \tag{8}$$

We now turn to a situation where firms compete simultaneously in the quality game. This assumption is in line with markets in which switching across quality grades is not very costly and, thus, firms cannot credible commit to a certain quality level. In this case, both firms choose their qualities simultaneously. This implies that both firms choose with knowledge of the other firm's best response function (as opposed to the actual choice as it is the case of the follower in the sequential game), and an equilibrium takes place when both firms are simultaneously playing their best response to the other firm's quality. As in the sequential case, we impose a no leapfrogging condition that guarantees a unique and stable equilibrium in pure strategies; and as in the sequential case the no leapfrogging condition results in one firm choosing a high quality product and the other firm choosing a lower quality. Which firm happens to choose the higher quality is inconsequential for efficiency and market equilibrium as both firms are otherwise homogeneous. We show in Appendix 1 that the no leapfrogging condition holds under the following assumptions:

ASSUMPTION 3. $0.75 < k_h < 1.54, \text{ if } k_l = 1,$

ASSUMPTION 4. $0.58 < k_l < 1.33, \text{ if } k_h = 1,$

Conditional on these assumptions, we use (5) to obtain best response function $v_h\,(v_l; k_h, k_l)$ and (6) to obtain best response function $v_l\,(v_k; k_h, k_l)$. We then find the intersection between these to compute the Nash Equilibrium (NE) qualities of the simultaneous game for the high- and low-quality firms:

$$v_h^n(k_h, k_l) \tag{9}$$

$$v_l^n(k_h, k_l) \tag{10}$$

Note that both quality and prices in the global sub-game perfect Nash equilibrium, for the simultaneous and sequential quality competition cases, depend upon consumers' perception of relative qualities, k_j, $j \in \{h, l\}$. The reaction of firms to misperception of relative qualities can be formally characterized by taking the derivative of the SPNE qualities and prices with respect to k_h or k_l, depending on the treatment we are discussing. We now turn to this issue.

2 Market equilibrium effects of quality misperception

2.1 Misperception on high-quality grades

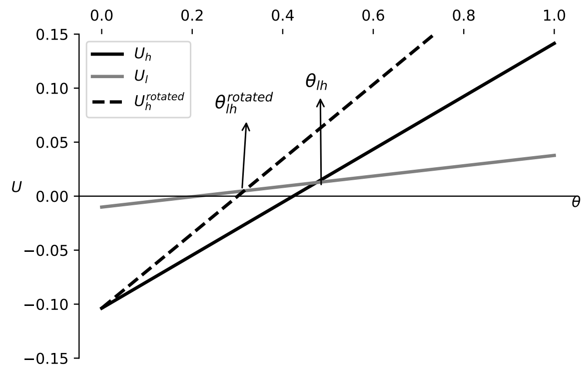
Misperception of the high-quality grade (i.e., deviations of k_h away from one) in the absence of misperception of the low-quality one ($k_l = 1$) unleashes multiple forces. A first-order effect is a shift in $D_h(v_h, v_l, p_h, p_l; k_h, k_l)$ which will prompt responses in qualities and subsequently on prices according to (5)-(8) in the case of sequential quality competition and (5), (6), (9), and (10) in the case of simultaneous quality competition. These changes in quality and prices alter the marginal consumers and effectively change the size of the market for low- and high-quality grades. We formally describe these changes in the following proposition:[2]

PROPOSITION 1. Under assumption 1 (in sequential quality competition), or under assumption 3 (in simultaneous quality competition) an increase in perceived quality of the high-quality grade, i.e. an increase in k_h from k_h^0 to k_h^1 where $k_h^0 < k_h^1$,

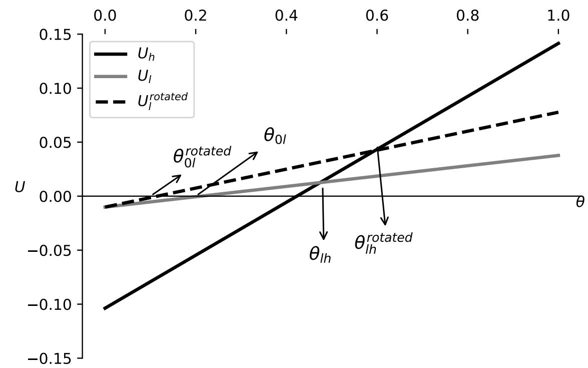
1. *increases quality of the high- and low-quality products;*
2. *increases prices of the high- and low-quality products;*
3. *increases quality-adjusted prices of the high- and low-quality products;*
4. *decreases market size of both products.*

[2] All proofs can be found in the appendix.

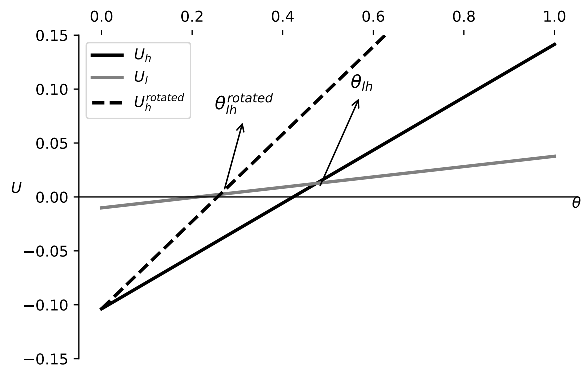
The mechanism underpinning results 1-4 in Proposition 1 is illustrated in Figure 1, and it applies to both sequential (under assumption 1) and simultaneous (under assumption 3) quality competition. The figure's horizontal axis represents consumers' WTP, while the vertical axis depicts values of indirect utility. As noted in our demand model, the intercepts of the indirect utility curves are equilibrium prices and the slope of the curves are determined by equilibrium qualities. Notice that the intersection between indirect utility curves marks the marginal consumer θ_{lh}, (i.e., the consumer that is indifferent between low- and high-quality grades), while the intersection between the horizontal axis and the low-quality indirect utility represents the marginal consumer θ_{0l} (i.e., the consumer indifferent between the low-quality grade and an outside option).



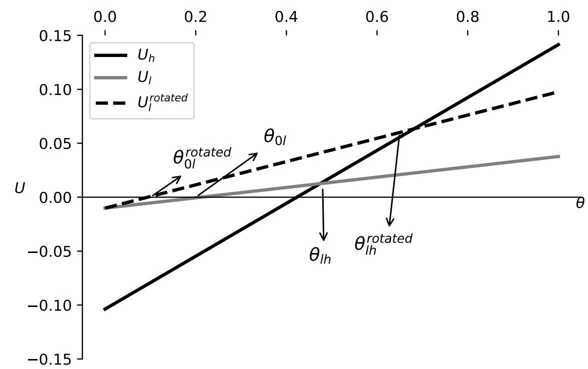
(a) Effect of overvaluation of high-quality product



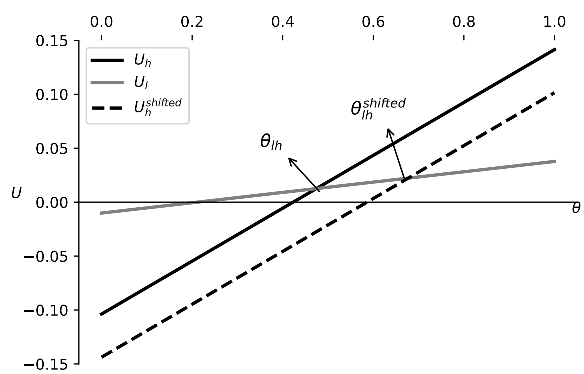
(b) Effect of overvaluation of low-quality product



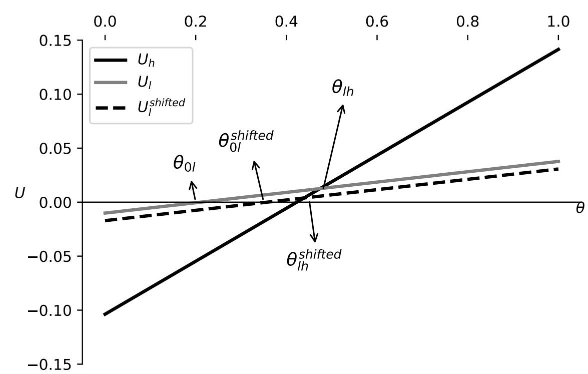
(c) Effect of quality increase in high-quality product



(d) Effect of quality increase in low-quality product



(e) Effect of price increase in high-quality product



(f) Effect of price increase in low-quality product

Figure 1: Effects of price and quality increase in the market of food labels. Each panel represents the effect of either prices or quality in marginal consumers, holding all else constant

Consumers' overvaluation of the high-quality grade increases the indirect utility of consumers buying the high-quality grade; and the increase is larger for consumers with a stronger preference for quality. This change is graphically represented in Figure 1a by a counterclockwise rotation of the indirect utility curve. The rotation shifts the marginal consumer θ_{lh} to the left, expanding the market for the high-quality grade. Overvaluation also strengthens the incentives to provide quality by the firm offering the high-quality product because it increases consumers' willingness to pay for higher quality. All else constant, a rise in the high-quality grade further rotates the corresponding indirect utility curve counterclockwise, as seen in panel 1c. As a result, marginal consumer θ_{lh} shifts further to the left and expands the market for the high-quality grade even more.

Following its best response function, the low-quality firm raises the quality of its product to capture market share from the high-quality firm (panel 1d); thus, the raise in quality of the high-quality product "pulls" the low-quality product up the quality spectrum. But, in conjunction with this, the follower also raises the price of its product and its margin which, as depicted in panel 1f, attenuates the gain in market share from increased quality. The low-quality firm will increase quality and price until the benefits and costs from increased margins, increased cost, and reduced market size are balanced out.

The forces depicted in Figures 1a-1f indicate that prices and qualities will raise in equilibrium as consumers increasingly overestimate the high-quality grade relative to the low-quality one (an increase in k_h). But these changes seem to trigger ambiguous effects on the level of quality-adjusted prices (i.e. the ratio of prices to qualities in equilibrium) and, consequently, the overall market size and welfare. Proposition 1 indicates that quality-adjusted prices raise enough to shrink the market for both products, but the market for the low-quality grade shrinks more.

2.2 Misperception on low-quality grades

Misperception of the low-quality grade (i.e., deviations of k_l away from one) in the absence of misperception of the high-quality one ($k_h = 1$) also unleashes multiple forces. But they differ in one crucial way from the effects of misperception on the high-quality grade; the overall effect of misperception of the low-quality grade varies according to the nature of quality competition. In sequential quality competition, the high-quality firm preempts the low-quality firm from increasing its own quality to gain market share. Overall, this curbs quality provision by both firms. On the other hand, under simultaneous quality competition, the high-quality firm does not have the ability to preempt the low-quality firm which spurs quality provision. Proposition 2 formally states the effects of misperception of the low-quality grade.

PROPOSITION 2. Under assumption 2 (sequential quality competition), or assumption 4 (simultaneous quality competition), an increase in perceived quality of the low-quality grade, i.e. an increase in k_l from k_l^0 to k_l^1 where $k_l^0 < k_l^1$, under sequential quality competition,

1. *lowers quality of the high-quality product and raises quality of the low-quality product;*
2. *lowers price of the high-quality product and raises price of the low-quality product ;*
3. *lowers quality-adjusted price of the high-quality product and raises quality-adjusted price of low-quality product;*
4. *increases market size for both products.*

Under simultaneous quality competition,

5. *raises quality of the high- and low-quality products;*
6. *lowers price of the high-quality product and raises price of the low-quality product;*
7. *lowers quality-adjusted price of the high-quality product and raises quality-adjusted price of the low-quality product;*
8. *increases market size for both products*

The mechanism underlying results in Proposition 2 is as follows. All else constant, an increase in k_l raises consumers' willingness to pay for the low-quality product (counterclockwise rotation of the consumer's indirect utility in figure 1b), expanding the size of this market (at the expense of markets for the unlabeled and high-quality labeled products). This strengthens the returns from quality provision by the low-quality firm . This firm raises the quality of its product which prompts an additional counterclockwise rotation of the indirect utility curve expanding the market for the low-quality grade, as shown in Figure 1d.

The ripple effects of these changes vary depending on the nature of quality competition. If quality competition is sequential the high-quality firm anticipates these changes and preemptively decreases its quality choice to protect its market share (it also benefits from a reduced cost of providing quality). But by lowering quality the high-quality firm also lowers its price in equilibrium, such that the quality-adjusted price of the high-quality product decreases. If quality competition is simultaneous, the high-quality firm cannot preempt the low-quality one, and both firms raise quality. Therefore, under simultaneous competition the increase in quality of the low-quality product "pushes" the high-quality product up the quality spectrum; this "push effect" does not take place under sequential competition.

While the reaction of the high-quality firm to overestimation of the low-quality product differs according to the nature of quality competition, the qualitative response of the low-quality firm to that reaction does not. In both cases the low-quality firm raises quality, price, and quality-adjusted price . The raise in quality of the low-quality product, in combination with consumers' stronger preference for it, intensifies price competition between products. As a result, the high-quality firm reduces its quality-adjusted price, expanding the size of its market, and possibly crowding out the low-quality product, especially in light of the increase in the quality-adjusted price of the low-quality product. However, the market for the low-quality product expands because the effects of overvaluation (figure 1b) and increased quality (figure 1d) overwhelm the negative effect of the price increase (figure 1f).

2.3 Mean-preserving misperception on high- and low-quality grades

Restraining misperception to the case where perceived average quality remains constant at the no-misperception scenario implies that any variation in misperception of the high-quality product is accompanied by a variation in misperception of the low-quality product in the opposite direction. Formally, we are concerned with combinations of k_h and k_l under which $\frac{k_h v_h^0 + k_l v_l^0}{2} = \frac{v_h^0 + v_l^0}{2}$, where v_l^0 and v_h^0 are equilibrium qualities without misperception, which means that $\frac{\partial k_l}{\partial k_h} = -\frac{v_h^0}{v_l^0}$. Such constant rate of variation in misperception implies that the effects of overvaluation (undervaluation) of quality misperception in the high-quality product will be augmented (or counterbalanced) by effects of undervaluation (overvaluation) of the low-quality product. The types of misperception considered so far necessarily alter the perceived average quality in the market which would have direct implications on market and, as we will see later, welfare effects. We now formally examine a type of misperception that does not alter the perceived average quality. Results are presented in proposition 3.

PROPOSITION 3. Under $0.95 < k_h < 1.05$*, a mean-preserving increase in perceived quality of the high-quality product; i.e. change from* k_h^0 *to* $k_h^1 > k_h^0$ *and change in* k_l^0 *by* $-\frac{v_h^0}{v_l^0}\left(k_h^1 - k_h^0\right)$,

1. *increases quality of the high-quality product, decreases quality of the low-quality product;*
2. *increases price of the high-quality product, decrease prices of the low-quality product;*
3. *increases quality-adjusted price of the high-quality product and decreases quality-adjusted price of the low-quality product;*
4. *decreases quantity consumed of both products in the market*

in both sequential and simultaneous quality competition.

The mechanisms underlying results in Proposition 3 resemble those previously discussed in Figure 1. An increase in k_h implies a counterclockwise rotation of the high-quality utility curve. The high-quality firm responds by raising quality and price (counterclockwise rotation followed by a downward shift of the utility curve), such that the quality-adjusted price increases. This reduces the market size of the high-quality product. In turn, the reduction in k_l implies a clockwise rotation of the low-quality indirect utility curve. The low-quality firm responds to misperception and the reaction of the high-quality firm by lowering price, quality, and quality-adjusted price. The combination of these responses with the demand shift from misperception results in a reduction in the size of the market for the low-quality product.

While our results indicate that changes in quality and the capacity to charge more per unit of quality mediate the distribution of welfare in the market, the direction of the effect of equilibrium displacement on consumers and firms, and thus, on the size of welfare, is unclear. The next section characterizes the effects of changes in misperception on welfare.

3 Welfare effects of quality misperception

3.1 Efficiency effects of quality misperception

The equilibrium displacement triggered by changes in consumers' quality misperception translates into changes in firms' profits and consumer surplus, altering market efficiency. In this section, we formally characterize the effect of consumers' misperception of quality on efficiency. We define total welfare as the summation of profit of the high-quality firm π_h from equation (1), profit of the low-quality firm π_l from equation (2), surplus of the segment of consumers purchasing the high-quality product (CS_h), and surplus of the segment of consumers purchasing the low-quality product (CS_l). Since we normalized indirect utility of the outside good to zero, consumer surplus from this segment of the market is zero. Therefore, welfare is defined as $W(v_h, v_l, p_h, p_l; k_h, k_l) = CS_h + CS_l + \pi_h + \pi_l$.

Changes in profits are straightforward to characterize from equations (1) and (2). In contrast, characterizing changes in consumer surplus is complicated by the fact that misperception causes a divergence between actual utility, defined as the one the consumer derives from the actual quality of the good, and the perceived utility, defined as the one the consumer derives from the perceived quality of the good. We follow the approach implemented in the literature (e.g., Glaeser and Ujhelyi, 2010; Brécard, 2014; Baksi et al., 2017) and evaluate consumer surplus based on the actual levels of quality provided. In other words, we remove the 'veil of ignorance' from consumers when computing their consumer surplus. Formally, instead of computing consumer surplus from buying the high-quality grades as $CS_h = \int_{\theta_{lh}}^{1} \theta k_h v_h - p_h \mathrm{d}\theta$, under misperception of high-quality, we compute it as $CS_h = \int_{\theta_{lh}}^{1} \theta v_h - p_h \mathrm{d}\theta$. For the same reason, we compute consumer surplus of low-quality consumers as $CS_h = \int_{\theta_{0l}}^{\theta_{lh}} \theta v_l - p_l \mathrm{d}\theta$.

Armed with our formal characterization of the equilibrium displacement triggered by misperception, we can compute equilibrium quality and prices under different levels of misperception that affect the perceived average quality in the market (Propositions 1-2). We subsequently insert these qualities and prices into expressions for π_h, π_l, CS_h, and CS_l, and add them up to compute welfare W. We formally state our results in Proposition 4.

PROPOSITION 4. Overvaluation (undervaluation) of either the high-quality grade or the low-quality grade; i.e. change from k_h^0 to $k_h^1 > k_h^0$ (from k_h^0 to $k_h^1 < k_h^0$) or from k_l^0 to $k_l^1 > k_l^0$ (from k_l^0 to $k_l^1 < k_l$), increases (decreases) welfare.

We follow the same procedure to compute welfare and surplus of consumer and producer segments in the case where misperception is mean-preserving, i.e. different levels of misperception that do not affect the perceived average quality in the market. The results from this process are formally stated in Proposition 5.

PROPOSITION 5. Under $0.95 < k_h < 1.05$, a mean-preserving increase (decrease) in perceived quality of the high-quality product; i.e. change from k_h^0 to $k_h^1 > k_h^0$ (k_h^0 to $k_h^1 < k_h^0$) and change in k_l^0 by $-\frac{v_h^0}{v_l^0}[k_h^1 - k_h^0]$ ($\frac{v_h^0}{v_l^0}[k_h^1 - k_h^0]$),

1. *increases (decreases) welfare under sequential quality competition*

2. *decreases (increases) welfare under simultaneous quality competition.*

A shock (misperception) increases welfare when it pushes the decentralized (market) resource allocation closer to a counterfactual benchmark that a social planner would choose. Understanding more precisely what reallocations push market equilibrium closer to that benchmark clarifies the forces underlying the effect of misperception on efficiency. We shed light on this issue by identifying a benchmark allocation against which the sub-game perfect Nash equilibrium allocation under misperception can be compared. It turns out that the first-best solution is one where only one product is offered in the market which is not direct comparable to our allocation. We constrain our analysis to the allocation of resources that maximizes efficiency but keeping the duopoly structure intact. This is a second-best solution in which the planner chooses quality, but firms compete in prices. Claim 1 compares the decentralized allocation of resources and the allocation under the second-best.

Claim 1. Irrespective of the nature of quality competition and in the absence of misperception, duopolist firms underprovide quality for both products relative to a second-best where the social planner chooses qualities and firms compete on prices conditional on those qualities.

This claim, in combination with our analysis of the market effects of misperception, help uncover the mechanism by which certain combinations of misperception and quality competition increase welfare. Our analysis of market equilibrium effects of misperception (Propositions 1-3) identifies several combinations of misperception and quality competition that result in higher qualities being offered in equilibrium. By Claim 1, this should push the market equilibrium closer to our second-best benchmark and, thus, increase welfare. However, results in Propositions 1-3 also indicate that increases in quality are often accompanied in equilibrium by a rise in quality-adjusted prices. This reduces the size of the market for labeled products and total surplus. The overall welfare effect will, therefore, depend on the relative strengths of these forces which in turn vary with the nature of misperception and quality competition.

Proposition 4 shows that overvaluation of either product keeping all else constant raises welfare, implying that efficiency gains from higher quality provision dominates efficiency losses from a reduced market size. This is because overvaluation of either product not only strengthens incentives for quality provision but also for consumers to purchase labeled products, limiting the reduction in market size associated with an increase in quality-adjusted prices. The same applies to a mean-preserving overvaluation of the high-quality product, as long as quality competition is sequential.

In contrast, Proposition 5 shows there is a situation in which average quality offered in the market drops as a result of misperception, and yet welfare increases with misperception. This happens under a mean-preserving undervaluation of the high-quality product and simultaneous quality competition. The average quality decreases because the reduction in quality of the high-quality product is larger than the rise in quality of the low-quality product. Furthermore, this reduces the perceived degree of differentiation between products which, in turn, galvanizes price competition. The intensified competition expands the size of the market for labeled products, and this expansion is strong enough to offset the reduction in average quality.

We conduct a numerical simulation that confirms our result that misperception can be welfare-enhancing, but reveals it is often local in nature (Figure 11, Appendix 2). When consumers overvalue the high-quality product only, misperception raises efficiency up to a point and decreases it afterwards. The same is true when consumers overvalue the low-quality product only and quality competition is sequential, and under a mean-preserving overvaluation of the high-quality product and quality competition is sequential. In these cases, efficiency gains from misperception vanish as misperception pushes the market qualities up and the decentralized solution approaches the second-best solution. By contrast, efficiency gains from overestimation of the low-quality product are global if quality competition is simultaneous. This is because qualities rise at a slower pace, approaching the second-best levels only asymptotically.

In sum, there are two channels through which misperception can enhance efficiency. First, it can enhance efficiency if it strengthens firms' incentives to increase quality offered in the market (quality effect). This tends to correct another market failure that takes place in the absence of misperception; the underprovision of quality that prevails under imperfect competition. Second, misperception can also enhance efficiency if it galvanizes competition and expands market size (market size effect). This corrects a different market failure that takes place in the absence of misperception; high markups and small market size that prevail under imperfect competition. Often these channels countervail each other; either firms provide higher quality but also increase quality-adjusted prices, or they provide lower qualities but lower quality-adjusted prices. Misperception raises efficiency when the quality effect is strong enough to dominate an increase in quality-adjusted prices, or when the market size effect is strong enough to dominate a reduction in qualities.

3.2 Distributional effects of quality misperception

While many types of misperception have similar qualitative effects on efficiency, they differ considerably on their impacts on profits of high- and low-quality firms, as well as surplus by consumer segment. We start by examining the distributional effects of misperception of the high-quality product. We present our results in the following proposition.

PROPOSITION 6. Under assumption 1 (in sequential quality competition) or assumption 3 (in simultaneous quality competition), overvaluation (undervaluation) of the high-quality product

1. *increases (decreases) profits of both firms*
2. *decreases (increases) surplus of consumers purchasing the high-quality product*
3. *increases (decreases) surplus of consumers purchasing the low-quality product*

under both simultaneous and sequential quality competition.

The market effects characterized in Proposition 1 shed light on these results. Overvaluation of the high-quality product prompts both firms to raise their quality, but also their quality-adjusted prices. An increase in quality-adjusted prices raise markup but at the expense of market size. However, the increase in qualities limits the contraction of the market size. As a result, both firms obtain higher profits. Moreover, both consumer segments are benefited by higher quality but the increase in quality-adjusted prices operates as a countervailing factor. The former effect dominates for consumers of the low-quality product raising their surplus, while the latter dominates for consumers of the high-quality product lowering their surplus. The effects of misperception of the low-quality product on distribution are described next.

PROPOSITION 7. Under assumption 2 (in sequential quality competition) or assumption 4 (in simultaneous quality competition), overvaluation (undervaluation) of the low-quality product

1. *decreases (increases) profit of the high-quality firm and increases (decreases) profit of the low-quality firm*
2. *increases (decreases) surplus of the high-quality consumer*
3. *decreases (increases) surplus of the low-quality consumer*

for simultaneous and sequential quality competition.

The distributional effects of misperception on the low-quality product (Proposition 7) stand in sharp contrast to those of misperception on the high-quality product (Proposition 6), despite the fact that they have similar qualitative effects on efficiency. Results presented in Proposition 2 shed light on this. They indicate that when consumers overvalue the low-quality product, both firms raise quality, but only the firm producing the low-quality product can raise quality-adjusted price. This explains why the low-quality firm obtains higher profits while the high-quality firm does not. It also explains changes in surplus by consumer segment.

Consumers of the high-quality product are benefited by a lower quality-adjusted price (and, in the case of simultaneous quality competition, a higher quality as well), while consumers of the low-quality product must pay a higher quality-adjusted price.

Finally, Proposition 8 summarizes the distributional effects of mean-preserving misperception.

PROPOSITION 8. Under $0.95 < k_h < 1.05$*, a mean-preserving increase in perceived quality of the high-quality product*

1. *decreases surplus of the high-quality consumer and increases surplus of the low-quality consumer.*
2. *increases profits of the high-quality firm and decreases profits of the low-quality firm under both simultaneous and sequential quality competition.*

The mechanism behind these results is similar to that in Propositions 6 and 7. The high-quality firm has the ability to increase its price more than proportionally to the quality supplied. This increases the high-quality firm's profit even though its market shrinks. Naturally, this also reduces surplus of consumers of the high-quality product. This is because there are less consumers purchasing the high-quality grade and those who still purchase it, pay a higher price per unit of quality. On the other hand, the low-quality firm lowers the quality-adjusted price of its product, but its market shrinks, nonetheless, due to consumers undervaluation of the low-quality grade. This reduces the low-quality firm's profit but also benefits those who purchase its product because they pay a lower price per unit of quality.

4 Implications for information-based policies

We discuss in this section how our insights can help refine policies aimed at curbing misperception. Propositions 4 and 5 identify conditions under which misperception raises or lowers efficiency. These conditions are expressed in terms of the nature of misperception and the nature of competition. The nature of misperception for certain product categories is routinely measured by empirical studies. These studies use experimental and observational data to determine whether consumers understand the information contained in labels and, as a result, whether they under- or over-estimate the quality of products offered in the market (e.g. McFadden and Lusk, 2018; Streletskaya et al., 2019; Villas-Boas et al., 2020). In turn, the nature of competition can be assessed based on observables, most prominently the size of sunk costs of providing higher quality relative to the marginal cost of production.

We found that policies that reduce misperception and, in doing so, lower consumers' valuation of certain grades relative to their competitors along the quality spectrum, reduce efficiency. At first glance, it seems counterintuitive that a policy-induced reduction in misperception can harm certain consumer segments and, more generally, efficiency. Our analysis shows that consumers that purchase the product whose quality is misperceived do benefit from more information. But they also show that the benefits of informing one consumer segment are dominated by the losses associated with weaker incentives to provide quality by both producers in the market. Previous studies indicate that the USDA's organic certification program is an example of such policies, because present rules are likely to induce consumers to undervalue non-organic relative to organic grades (e.g. Streletskaya et al., 2019). In this case, it may be advisable to explicitly certify gradations of organic below 95%, instead of opting for a more binary approach (the USDA organic seal is only granted to above 95%). Other prominent examples are mandates to disclose in labels the presence of attributes that reduce consumers' valuation of products. These include mandates to disclose the presence of genetically modified organisms (California Proposition 37, Vermont Act 120, and the National Bioengineered Food Disclosure Standard). In these instances, it may be advisable to certify the presence of a positive attribute (non-GMO or no Trans Fat) instead of requiring disclosure of a negative attribute (contains GMOs or contains Trans Fat).

We also found that a policy in which a reduction in misperception implies raising consumers' valuation of the high-quality product and lowering consumers' valuation of the low-quality product may also hamper efficiency in markets where sunk costs of providing quality are relatively small. In this type of market, the benefits of informing both consumer segments (and reducing their misperception) are dominated by losses associated with weaker price competition, conditional on equilibrium qualities. This is because the policy prompts firms to increase product differentiation, softening competition. Statutory rules regulating information on nutritional contents fit this case. First, sunk costs of providing quality are relatively low which makes this market resemble one with a simultaneous quality competition. Moreover, FDA rules require disclosing the presence of trans-fat which induces undervaluation of these products (Villas-Boas et al., 2020), and allow for label stacking such as organic and non-GMO which induces overvaluation of these products (McFadden and Lusk, 2018). In this case, our analysis indicates that the government could potentially raise efficiency by not disallowing redundant labels and eliminating mandates to disclose a negative attribute. Interestingly, not informing consumers of a negative attribute would protect them by providing the low-quality firm to raise quality.

5 Conclusions

Governments around the world have and continue to implement rules that regulate the information that can, cannot, and must be included in food labels. The main goal of these policies is to curb consumers' misperception (a pervasive phenomenon according to empirical evidence) thereby, so the argument goes, helping them make better choices and raising welfare. Therefore, information-based policies are based on the premise that misperception harms consumers and, perhaps, efficiency. We examine the validity of this premise by studying the market and welfare effects of quality misperception by consumers.

We find that the premise underpinning information-based policies is often erroneous, and that misperception can in fact benefit consumers and enhance efficiency due to firms' strategic reactions to it. However, the relationship between misperception and welfare hinges upon the direction of misperception, where misperception occurs along the quality spectrum, and the nature of quality competition among firms. This underscores the usefulness of the framework we develop in this study. In contrast to previous contributions to this literature, our framework is general enough to consider a range of misperception types that seem supported by empirical evidence, as well as different types of quality competition depending on firms' ability to commit to a quality grade, preempting competitors to enter that market segment. On the other hand, our framework is also parsimonious enough to formally characterize combinations of misperception and quality competition under which misperception enhances efficiency.

One notable limitation of our analysis is that we do not consider potential entry of more firms along the quality spectrum, nor do we consider multi-product firms. Such extensions could fundamentally change our insights, though predicting the nature of those changes requires more than simple intuition. A closely related limitation is that we do not consider horizontal differentiation along with vertical differentiation. This could also change the nature of our results as it may induce strategies like fighting brands and product line pruning, strategies that have been studied in markets without misperception (Shen et al., 2016). We believe a promising extension of this research is to develop a framework to consider these, more complicated trading environments.

Moreover, we do not offer in this study a fully-fledged strategy to make our theoretical analysis empirically operational as such an endeavor exceeds the boundaries of this study. This seems like a useful extension of our analysis. A significant challenge to this task is to empirically measure misperception, as well as the plausible effects of new regulations on misperception. This is challenging because misperception is the difference between consumer's perceived quality of a product, and the quality the consumer would perceive had they had full information and understanding of credence attributes in that product. Both of these measures are typically unobservable to regulators, though maybe obtainable through randomized controlled experiments. Yet, in this study we are able to provide what we believe are useful guiding principles in the section titled “Implications for information-based policies”.

Appendix 1 - No leapfrogging conditions

We use equations 11- 16 throughout appendix 1 and 2. Equations 11 and 12 are derived by substituting equilibrium prices $p_i^*(v_h, v_l; k_h, k_l)$, $i \in \{h, l\}$ in the revenue functions. For misperception on high-quality, we set $k_l = 1$ in the equations below; whereas for misperception on low-quality, we set $k_h = 1$ in the equations below.

$$R_h = \frac{4k_h^2 v_h^2 (k_h v_h - k_l v_l)}{(4k_h v_h - k_l v_l)^2}. \tag{11}$$

$$R_l = \frac{k_h k_l v_h v_l (k_h v_h - k_l v_l)}{(4k_h v_h - k_l v_l)^2}. \tag{12}$$

$$\frac{\partial R_h}{\partial v_h} = \frac{4k_h^2 v_h (4k_h^2 v_h^2 - 3k_h k_l v_h v_l + 2k_l^2 v_l^2)}{(4k_h v_h - k_l v_l)^3} > 0 \text{ if } \frac{v_l}{v_h} < \frac{4k_h}{7k_l}. \tag{13}$$

$$\frac{\partial R_h}{\partial v_l} = -\frac{4k_h^2 k_l v_h^2 (2k_h v_h + k_l v_l)}{(4k_h v_h - k_l v_l)^3} < 0, \text{ if } \frac{v_l}{v_h} < \frac{4k_h}{7k_l}. \tag{14}$$

$$\frac{\partial R_l}{\partial v_l} = \frac{k_h^2 k_l v_h^2 (4k_h v_h - 7k_l v_l)}{(4k_h v_h - k_l v_l)^3} > 0 \text{ if } \frac{v_l}{v_h} < \frac{4k_h}{7k_l}. \tag{15}$$

$$\frac{\partial R_l}{\partial v_h} = \frac{k_l k_h v_h^2 (2k_h v_h + k_l v_l)}{(4k_h v_h - k_l v_l)^3} > 0, \text{ if } \frac{v_l}{v_h} < \frac{4k_h}{7k_l}. \tag{16}$$

Armed with these definitions, we start by showing conditions under which the no leapfrogging condition holds for different treatments.

Sequential competition under misperpcetion of high-qualty grade

First, we explain the notation used in this appendix. Let v represent quality, let the superscript $\{L, F\}$ stand for leader and follower firm, respectively, and let the subscript $\{l, h\}$

refers to low and high quality, respectively. We will use the following convention: a given function $K(x, y)$ depends on the high-quality and low-quality level chosen by firms, such that x is always the lower level of quality and y is the higher level of quality. For example, the function $K_h^L(x, y)$ refers to the high-quality (subscript h) leader's (superscript L) $K(\cdot)$ function, under high-quality choice y and low-quality choice x. Similarly, the notation for the low-quality follower in this example would be $K_l^F(x, y)$, where it chooses quality level x, and y is the leader's quality choice.

We show that there exists a unique equilibrium in pure strategies for the sequential game over a range of misperception parameters, such that the leader's quality in equilibrium is higher than the follower's quality in equilibrium. The set of conditions that guarantee such equilibrium in pure strategies are known as the no-leapfrogging conditions (Motta, 1993; Lehmann Grube, 1997).

To understand the no leapfrogging conditions, we first discuss the leader's possible decisions, following Lehmann Grube (1997). The leader has three options: i) it can choose a low-quality level v_l^L that forces the follower to best-respond with a higher-quality level $v_h^F = b(v_l^L)$; ii) the leader can choose the high quality level v_h^L that guarantees that the follower best-responds with a lower-quality level $v_l^F = h(v_h^L)$; iii) leader can choose a level of quality $\hat{v}$ such that the follower is indifferent between choosing a higher $\hat{v}_h$ or lower $\hat{v}_l$ quality level than $\hat{v}$. To show no leapfrogging conditions in sequential games, we must show that, over a range of misperception parameters, (1) the leader makes higher profits by positioning as high-quality firms rather than lower-quality firm and (2) the high-quality leader makes higher profits than lower-quality follower in equilibrium. This is formally defined in equations 17 and 18:

$$\pi_l^L = R_l^L(v_l^L, v_h^F; k_h) - C(v_l^L) < R_h^L(v_l^F, v_h^L; k_h) - C(v_h^L) = \pi_h^L \tag{17}$$

$$\pi_l^F = R_l^F(v_l^s, v_h^s; k_h) - C(v_l^s) < \pi_h^L = R_h^L(v_l^s, v_h^s; k_h) - C(v_h^s) \tag{18}$$

where v_h^s is the maximum between $\hat{v}$ and v_h^L. Equation 17 guarantees that the leader's profit under its best high-quality choice, conditional of follower best responding with a low-quality choice, strictly dominates the leader's profit under its best low-quality choice, conditional on the follower best responding with a high-quality choice. Equation 18 guarantees that the high-quality leader has no incentive to deviate from its best high-quality choice, say choosing a quality that is closer to the follower's low-quality choice, because the leader can always make higher profits than the follower. Together, condition (1) states that the leader will force the follower to best-respond with lower-quality and once the leader chooses high-quality, condition (2) guarantees that the leader's choice yields the highest profit among the firms, which implies that there is no incentive to deviate from it. These are the no-leapfrogging conditions.

LEMMA A1. Conditional on a misperception parameter, (1) the best low-quality choice by the leader in sequential games, v_l^L, is higher than the optimal low-quality choice in simultaneous games, v_l^n; (2) the follower's high-quality best-response to (1), $v_h^F(v_l^L) = b(v_l^L)$, is higher than the optimal high-quality choice in simultaneous games, v_h^n.

Proof: Suppose the leader chooses low quality. The necessary condition for optimality must satisfy the leader's first-order conditions of the sequential game:

$$d\pi_l^L / dv_l^L = 0, \tag{19}$$

which can be written $\partial R_l^L/\partial v_l^L + (\partial R_l^L/\partial v_h^F) b'\left(v_l^L\right) = dC(v_l^L)/dv_l^L$. Notice that the first-order condition of the sequential and simultaneous game for the follower is $\partial R_h^F/\partial v_h^F - dC^L(v_h^F)/dv_h^F = 0$. We can totally differentiate the follower's first-order condition with respect to v_l^L and rearrange the terms to obtain:

$$b^{'}(v_l^L) = \underbrace{\frac{\partial \frac{\partial R_h^F}{\partial v_h^F}}{\partial v_l^L}}_{>0} \Bigg/ \left(\underbrace{\frac{\partial \frac{\partial C_h}{\partial v_h^F}}{\partial v_h^F}}_{>0} - \underbrace{\frac{\partial \frac{\partial R_h^F}{\partial v_h^F}}{\partial v_h^F}}_{<0} \right) > 0, \tag{20}$$

which implies $b'(v_l^L) > 0$. Notice that $\partial R_l^L / \partial v_h^F > 0$. Also, notice that the necessary conditions for a solution for a simultaneous quality game would imply $\partial R_l^L / \partial v_l^L = dC^L(v_l^L)/dv_l^L$. Thus, by comparing first-order conditions of the sequential and simultaneous games, we know that $v_l^L > v_l^n$, where v_l^n is the solution for a simultaneous game. Since best responses are monotonic, we know that the follower chooses $v_h^F > v_h^n$ at the optimal values.

LEMMA A2. Conditional on a misperception parameter, (1) the best high-quality choice by the leader in a sequential game, v_h^L, is lower than the optimal high-quality choice in simultaneous games, v_h^n; (2) the follower's low-quality best response (1), $v_l^F(v_h^L) = h(v_h^L)$, is lower than the optimal high-quality choice in simultaneous games, v_l^n.

Proof: Assume that the leader acts as the high-quality firm by choosing quality level v_h^L. Under these circumstances, the follower would best respond with $v_l^F = h\left(v_h^L\right)$. Now, the necessary condition for optimality must satisfy the first-order conditions of the sequential game, such that:

$$\frac{\partial \pi_h^L}{\partial v_h^L} = \frac{\partial R_h^L}{\partial v_h^L} + \frac{\partial R_h^L}{\partial v_l^F} h^{'}(v_h^L) = \frac{dC^L(v_h^L)}{dv_h^L}. \tag{21}$$

Notice that $\partial R_h^L / \partial v_l^F < 0$. One can check that $h'(v_h^F) > 0$ using the same procedure used in Lemma A1. Comparing first-order conditions for the sequential and simultaneous games, we can check that $v_h^L < v_h^n$, where v_h^n is the solution of the simultaneous quality game. Since the best response function is monotonic, we know that the follower chooses $v_l^F < v_l^n$.

LEMMA A3. Conditional on a misperception parameter, we can state $v_h^F > v_l^L > \widetilde{v}_l$, where $\widetilde{v}_l$ is the best response of the follower in case the leader chooses v_h^F.

Proof: Assume that the leader acts as the high-quality follower and chooses v_h^F. The follower would respond with $\widetilde{v}_l = h(v_h^F)$. The necessary condition of the sequential game are:

$$\frac{\partial \pi_h^L}{\partial v_h^F} = \frac{\partial R_h^L}{\partial v_h^F} + \frac{\partial R_h^L}{\partial \widetilde{v}_l} h^{'}(v_h^F) = \frac{dC^L(v_h^F)}{dv_h^F}. \tag{22}$$

Again, we get that $\widetilde{v}_l < v_l^n$. Then, using the results from Lemma A1 and Lemma A2, and the necessary conditions for the sequential game when the leader acts as the higher-quality follower, we can state the following relationship: Let $v_h^F = z$, where $z \in \{R\}_{++}$. Then, $v_h^F = z > v_l^L = az > \widetilde{v}_l = \hat{a}z$, for $1 > a > \hat{a} > 0$.

LEMMA A4. High-quality leader's profit is bounded below by the quality choice of v_h^F.

Proof: Assume that, all else constant, the leader's profit level can increase by choosing a v_h, such that $v_h < v_h^F$. Notice that the leader's profit is decreasing in v_l. Since the follower's quality best response is monotonic, i.e., $v_l'(vh) > 0$, a deviation from v_h^F to v_h can only increase the leader's profit. Notice that by Lemma A1 and A2, a deviation from the leader's quality choice cannot be higher than v_h^F without violation of the FOCs. Thus, the high-quality leader's profit is bounded below by v_h^F.

Next, we will show a set of misperception parameter that is sufficient for condition (1) to hold. Again, we are not deriving necessary conditions on an interval of misperpcetion such that condition (1) holds; rather, we are finding a sufficiently large interval of misperception by which we can discuss the effects of misperception on market outcomes, welfare, and distribution.[3]

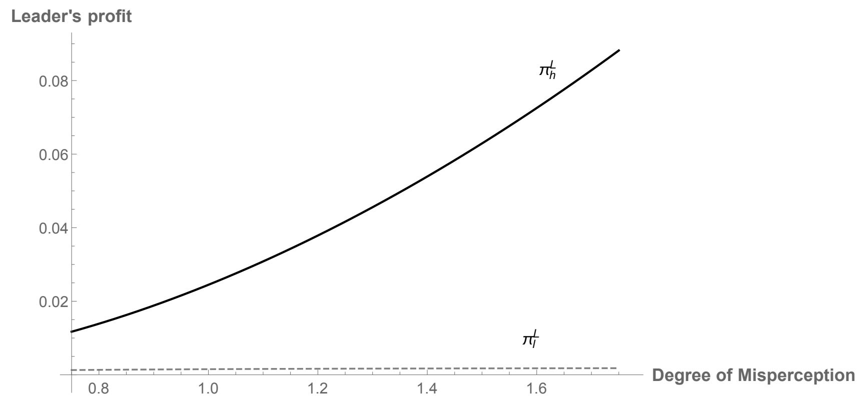
Suppose the leader chooses low quality v_l^L (best low-quality for leader), and follower best respond with v_h^F, then $\pi_l^L(v_l^L, v_h^F; k_h)$ is the low-quality leader's profit. By Lemma A4, we know that the profit of the high-quality leader is bounded below when the leader chooses v_h^F

[3]One can find the largest possible interval of k_h by which the no leapfrogging holds by numerically checking whether equations 17 and 18 hold for incremental values of k_h. The implementation of such algorithm is tedious and likely not to yield any interesting additional insight from the implementation discussed in the appendix. While we eventually resort to solutions under quadratic costs in this appendix, the reader can check that other convex cost structures can be used to the same qualitative conclusions.

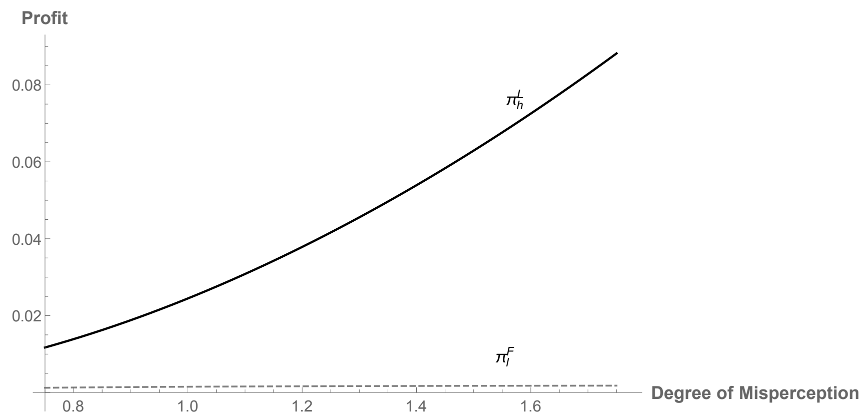
and the follower best respond with the low-quality level $\widetilde{v}_l^F$, which implies that the leader's profit is given by $\pi_h^L(\widetilde{v}_l^F, v_h^F; k_h)$. Lehmann Grube (1997) shows that under no misperception, the leader always chooses high-quality. We will check that there exists a interval of misperception parameters around the no misperception case for which a leader prefers to choose high-quality. To do that, it is sufficient to find the set interval of misperception parameters that guarantee that the lower bound profit level of the high-quality leader is above the profit level associated with the best low-quality choice by the leader. Formally, we want a range of misperception parameters that guarantees:

$$\pi_h^L\left(\widetilde{v}_l^F,\ v_h^F; k_h\right) = R_h^L\left(\widetilde{v}_l^F,\ v_h^F; k_h\right) - C\left(v_h^F\right) > R_l^L\left(v_l^L,\ v_h^F; k_h\right) - C\left(v_l^L\right) = \pi_l^L(v_l^L,\ v_h^F; k_h) \tag{23}$$

We can show that inequality 23 holds for $0.75 < k_h < 1.75$ – we call it Assumption 1 – by substituting the revenue functions 11 and 12 in equation 23 and assuming quadratic costs. Under Assumption 1, equation 17 holds, as shown in Figure 2a Within the range $0.75 < k_h < 1.75$, one can check numerically that equation 18 (leader's profit is higher than follower's profit) also holds for quadratic costs, as depicted in Figure 2b. However, more generally, one can also notice that condition (2) is always satisfied for a k_h range in which a high-quality firm makes higher profits than the lower quality firm for the optimal solution of the simultaneous quality game. That is because (i) by Lemma A2, $v_h^L < v_h^n$, (ii) profit level of the low-quality firm is increasing in high- and low-quality levels, and (iii) the best response of the follower is monotonically increasing in the levels of leader's quality choice. Thus, if the follower's profit is smaller than the leader's profit under $\{v_h^n,\ v_l^n\}$, then (i)-(iii) guarantee that the leader still earns higher profit than the follower under $\{v_h^L, v_l^F\}$. Lastly, under quadratic costs, the leader's quality level that leaves the follower indifferent to best respond with higher or lower quality, $\hat{v}$, is below v_h^L. All together, these conditions implies that v_h^L is enough to prevent leapfrogging.



(a) Effect of misperception, k_h



(b) Effect of misperception, k_h

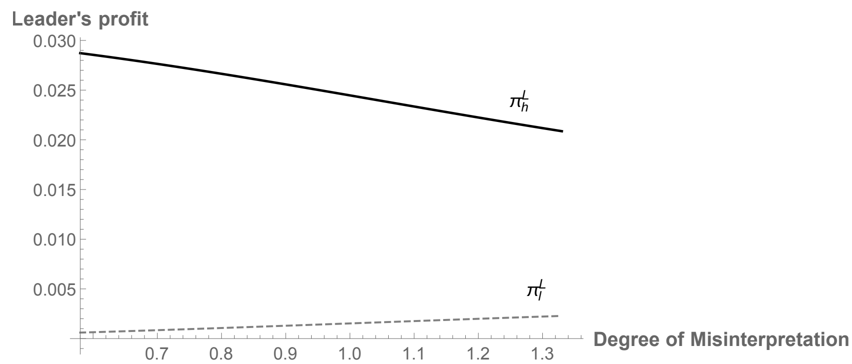
Figure 2: Profit variations with variations in misperception k_h. Panel (a) shows variation in profits of the leader when it chooses best high-quality quality and it chooses best low-quality quality. Panel (b) shows variation in profits when the leaders positions as the high-quality firm and follower positions as the low-quality firm.

Sequential competition under misperception of low-quality grade

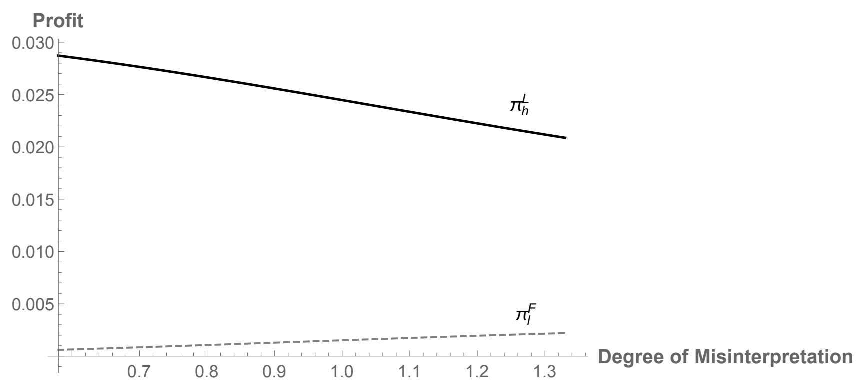
The argument follows exactly the same structure of the case sequential, k_h. Lemmas A1-A4 hold for the case in which misperception is in the low-quality k_l. Similarly to equation 23, we can find a sufficient large interval by which the leader prefers to position itself as the high-quality firm. This is represented is represented by:

$$R_h^L\left(\widetilde{v}_l^F,\ v_h^F; k_l\right) - C\left(v_h^F\right) > R_l^L\left(v_l^L,\ v_h^F; k_l\right) - C(v_l^L) \tag{24}$$

We can show that inequality 24 holds for $k_l \in \{0.58, 1.33\}$ -– call it assumption 2. The rest of the proof is shown in figure 3, and it uses uses the same arguments as the case of misperception in high-quality grades.



(a) Effect of misperception, k_l



(b) Effect of misperception, k_l

Figure 3: Profit variations with variations in misperception k_l. Panel (a) shows variation in profits of the leader when it chooses best high-quality quality and it chooses best low-quality quality. Panel (b) shows variation in profits when the leaders positions as the high-quality firm and follower positions as the low-quality firm.

Mean-preserving misperception under sequential quality choice

We now consider the presence of both k_h and k_l. However, to guarantee that the average perception of quality does not change with changes in relative misperception (i.e. same iso-misperception line), we impose a relationship between k_h and k_l. We let average perceived quality under no misperception $\bar{kv}$ follow equation 25.

$$k_h v_h^0 + k_l v_l^0 = \bar{kv}, \tag{25}$$

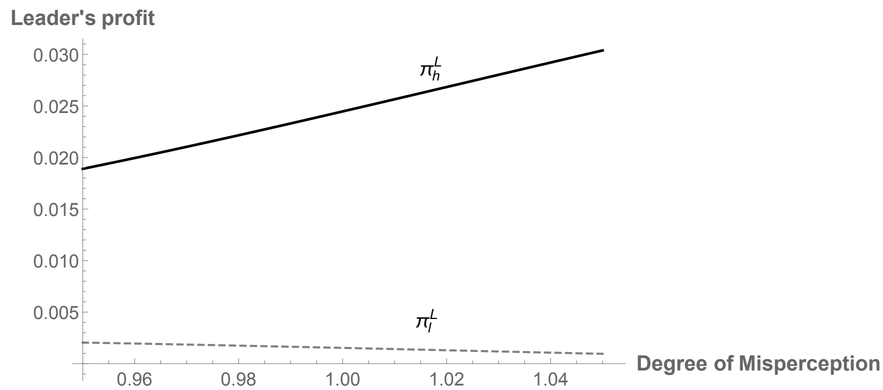
where v_h^0 and v_l^0 are the optimal qualities in equilibrium under no misperception. We can solve equation 25 for k_l to obtain equation 26.

$$k_l = (\bar{kv} - k_h v_h^0)/v_l^0, \tag{26}$$

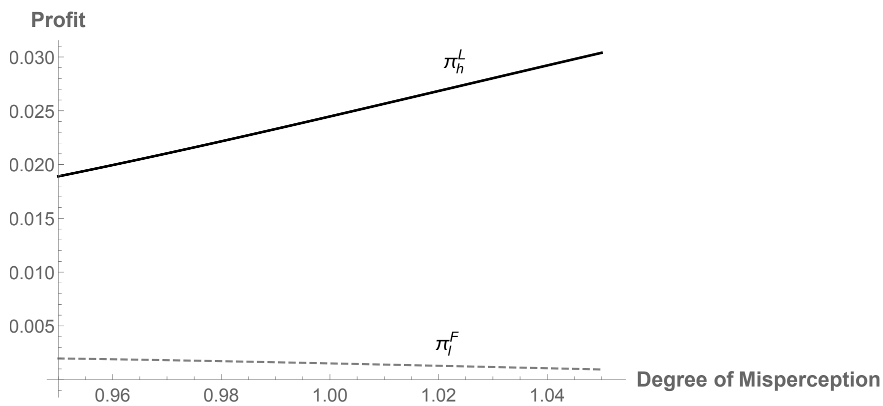
To check whether the leader would prefer to be the high-quality firm we must check equation 27 below.

$$R_h^L\left(\widetilde{v}_l^F, v_h^F; k_h, k_l\right) - C\left(v_h^F\right) > R_l^L\left(v_l^L,\ v_h^F; k_h, k_l\right) - C(v_l^L) \tag{27}$$

Substituting equation 27 into equation 26 under quadratic costs, one can check that $k_h \in \{0.95, 1.05\}$ satisfies inequality 27. Numerical solutions also reveal that the leader makes higher profit than the follower when it is the high-quality firm, which completes the argument for no leapfrogging conditions. This can be check in figure 4:



(a) Effect of misperception, $k_h(k_l)$



(b) Effect of misperception, $k_h(k_l)$

Figure 4: Profit variations with variations in misperception $k_h(k_l)$. Panel (a) shows variation in profits of the leader when it chooses best high-quality quality and it chooses best low-quality quality. Panel (b) shows variation in profits when the leaders positions as the high-quality firm and follower positions as the low-quality firm.

Simultaneous competition under misperception of high-quality grade

Using quadratic costs under assumption 1, we can check the no-leapfrogging conditions using the definition by Motta (1993). We need to check that, for a given misperception parameter, the low-quality firm has no incentive to leapfrog the high-quality firm and vice-versa. The no-leapfrogging condition in simultaneous games entails (1) that the low-quality firm has no incentive to become the high-quality producer by choosing a value higher than the optimal quality chosen by the high-quality firm; and (2) that the high-quality firm has no incentive to become the low-quality producer by choosing a value lower than the optimal quality chosen by the low-quality firm. Formally, condition (1) can be checked by:

$$\pi_h^{leap}\left(v_h^n, v_h^{leap}; k_h\right) < \pi_l^n\left(v_l^n,\ v_h^n; k_h\right), \tag{28}$$

where $v_h^{leap} = \beta \times v_h^n$, where $\beta > 1$, is the value chosen by the leapfrogging low-quality that becomes high-quality. Similarly, condition (2) can be found by:

$$\pi_l^{leap}\left(v_l^{leap},\ v_l^n; k_h\right) < \pi_h^n\left(v_l^n,\ v_h^n; k_h\right), \tag{29}$$

where $v_l^{leap} = \beta \times v_h^n$, where $0 < \beta < 1$, is the value chosen by the leapfrogging high-quality that becomes low-quality. Both inequalities hold when we restrict the misperception parameter to $0.75 < k_h < 1.53$.

Simultaneous competition under misperception of low-quality grade

We use the same procedure described in simultaneous quality competition under misperception of high-quality grade above. Condition (1) can be checked by:

$$\pi_h^{leap}\left(v_h^n, v_h^{leap}; k_l\right) < \pi_l^n\left(v_l^n,\ v_h^n; k_l\right), \tag{30}$$

where $v_h^{leap} = \beta \times v_h^n$, where $\beta > 1$, is the value chosen by the leapfrogging low-quality that becomes high-quality. Similarly, condition (2) can be found by:

$$\pi_l^{leap}\left(v_l^{leap},\ v_l^n; k_l\right) < \pi_h^n\left(v_l^n,\ v_h^n; k_l\right), \tag{31}$$

where $v_l^{leap} = \beta \times v_h^n$, such that $0 < \beta < 1$, is the value chosen by the leapfrogging high-quality that becomes low-quality. Both inequalities hold when we restrict the misperception parameter to $0.58 < k_l < 1.33$.

Simultaneous competition under mean-preserving misperception

Using quadratic cost and under equation 25 (same iso-misperception curve as in perfect information), we can check the no-leapfrogging conditions using the same procedure described for the cases under simultaneous k_h and simultaneous k_l. Thus, we need to check whether equations 32 and 33 hold:

$$\pi_h^{leap}\left(v_h^n, v_h^{leap}; k_h(k_l)\right) < \pi_l^n\left(v_l^n,\ v_h^n; k_h(k_l)\right), \tag{32}$$

where $v_h^{leap} = \beta \times v_h^n$, where $\beta > 1$, is the value chosen by the leapfrogging low-quality that becomes high-quality. Similarly, condition (2) can be found by:

$$\pi_l^{leap}\left(v_l^{leap},\ v_l^n; k_h(k_l)\right) < \pi_h^n\left(v_l^n,\ v_h^n; k_h(k_l)\right), \tag{33}$$

where $v_l^{leap} = \beta \times v_h^n$, where $0 < \beta < 1$, is the value chosen by the leapfrogging high-quality that becomes low-quality. As in the sequential case, we impose $0.95 < k_h < 1.05$ and check that conditions holds.

Appendix 2

Proof of Proposition 1.

Misperception in high-quality, k_h, under sequential quality choice.

We show, for the sequential quality choice case, that 1) quality choices increase, 2) that quantity consumed of products decrease, 3) that prices increase. First, notice that the first-order condition of the sequential game for the follower is $\frac{\partial R_l(v_h, vl(vh); k_h)}{\partial v_l} - \frac{dC^l(v_l(v_h); k_h)}{dv_l} = 0$. We can totally differentiate this expression with respect to v_h and rearrange the terms to obtain the variation of the best response function, v_l^{BR}:

$$\frac{dv_l^{BR}(v_h;k_h)}{dv_h} = \underbrace{\frac{\partial \frac{\partial R_l}{\partial v_l}}{\partial v_h}}_{>0} \Bigg/ \left(\underbrace{\frac{\partial \frac{\partial C_l}{\partial v_l}}{\partial v_l}}_{>0} - \underbrace{\frac{\partial \frac{\partial R^l}{\partial v_l}}{\partial v_l}}_{<0} \right) > 0. \tag{34}$$

Second, notice that the marginal revenue functions are homogeneous of degree 0. Thus, we can state:

$$v_l \frac{\partial \frac{\partial R_l}{\partial v_l}}{\partial v_l} + v_h \frac{\partial \frac{\partial R_l}{\partial v_l}}{\partial v_h} = 0 \tag{35}$$

$$\frac{v_l}{v_h} = \frac{\partial \frac{\partial R_l}{\partial v_l}}{\partial v_h} \Bigg/ - \frac{\partial \frac{\partial R_l}{\partial v_l}}{\partial v_l}, \tag{36}$$

From equations 34 and 35, we know that $v_l/v_h < dv_l^{BR}(v_h;k_h)/dv_h$. Notice that $v_l/v_h < 1$, by definition. Thus, we can state:

$$1 > \frac{v_l}{v_h} > \frac{dv_l^{BR}(v_h;k_h)}{dv_h} \tag{37}$$

We are going to use monotone comparative statics to sign the effects of k_h in qualities. For monotone comparative statics, we need to show that strategies are complements and the exogenous parameter has increasing differences with the necessary conditions of the game. We can check that $\{v_h, v_l\}$ are strategic complements for both firms. This is done by checking $\frac{\partial^2 \pi_l}{\partial v_l \partial v_h} > 0$ and $\frac{\partial^2 \pi_h}{\partial v_h \partial v_l} > 0$.

$$\frac{\partial^2 \pi_l}{\partial v_l \partial v_h} = \frac{2k_h^2 v_h v_l (8k_h v_h + 7v_l)}{(4k_h v_h - v_l)^4} > 0, \tag{38}$$

which immediately guarantees strategic complementarity between v_l and v_h for the follower. We do the same operation for the leader:

$$
\begin{aligned}
\frac{\partial^2 \pi_h}{\partial v_h \partial v_l} &= \frac{\partial \frac{\partial R_h(v_h,v_l)}{\partial v_h}}{\partial v_l} + \frac{\partial \frac{\partial R_h(v_h,v_l)}{\partial v_l}}{\partial v_l} \frac{dv_l^{BR}(v_h;k_h)}{dv_h} \\
\frac{\partial^2 \pi_h}{\partial v_h \partial v_l} &= \frac{8k_h^2 v_h v_l (5k_h v_h + v_l)}{(4k_h v_h - v_l)^4} - \frac{8k_h^2 v_h^2 (5k_h v_h + v_l)}{(4k_h v_h - v_l)^4} \frac{dv_l^{BR}(v_h;k_h)}{dv_h} \\
\frac{\partial^2 \pi_h}{\partial v_h \partial v_l} &= (v_l - \frac{dv_l^{BR}(v_h;k_h)}{dv_h} v_h) \frac{8k_h^2 v_h (5k_h v_h + v_l)}{(4k_h v_h - v_l)^4},
\end{aligned} \tag{39}
$$

which is positive by equation 37. Then, we check for increasing differences in the policy parameter k_h. Again, starting by the follower:

$$
\frac{\partial^2 \pi_l}{\partial v_l \partial k_h} = \frac{2k_h v_h^2 v_l (8k_h v_h + 7v_l)}{(4k_h v_h - v_l)^4} > 0. \tag{40}
$$

The same operation for the leader to obtain a large expression (suppressed for sake of space) that is positive under Assumption 1:

$$
\frac{\partial^2 \pi_h}{\partial v_h \partial k_h} = \frac{\partial \frac{\partial R_h}{\partial v_h}}{\partial k_h} + \frac{\partial}{\partial k_h} \left(\frac{\partial R_h(v_h, v_l; k_h)}{\partial v_l} \frac{dv_l^{BR}(v_h;k_h)}{dv_h} \right) \tag{41}
$$

$$
= \frac{\partial \frac{\partial R_h}{\partial v_h}}{\partial k_h} + \frac{\partial \frac{\partial R_h}{\partial v_l}}{\partial k_h} \frac{dv_l^{BR}(v_h;k_h)}{dv_h} + \frac{\partial \frac{dv_l^{BR}(v_h;k_h)}{dv_h}}{\partial k_h} \frac{\partial R_h}{\partial v_h} \tag{42}
$$

$$
> 0 \text{ if } \frac{v_l}{v_h} < \frac{4k_h}{7} \tag{43}
$$

Thus, the strategic complementary between firms strategy and the increasing differences in the policy parameter k_h leads to:

$$
\frac{dv_h^s}{dk_h} > 0, \tag{44}
$$

$$
\frac{dv_l^s}{dk_h} > 0, \tag{45}
$$

This proves that qualities increase with the misperception parameter k_h under sequential quality competition, as stated in Proposition 1.

We check the effects of an increase in misperception on the size of the market of high-quality products next. First, we check the effect of an increase in k_h on the marginal consumer indifferent between high quality and low-quality products, $\theta_{lk}(v_k^*(kh), v_l^*(kh); k_h)$, as in equation 46

$$\begin{aligned}\frac{d\theta_{lh}}{dk_h} &= \frac{\partial\theta_{lh}}{\partial k_h} + \frac{\partial\theta_{lh}}{\partial v_h}\frac{dv_h}{dk_h} + \frac{\partial\theta_{lh}}{\partial v_l}\frac{dv_l}{dk_h} \\ &= \frac{2v_h v_l}{(4v_h k_h - v_l)^2} + \left(v_l\frac{dv_h}{dk_h} - v_h\frac{dv_l}{dk_h}\right)\frac{2k_h}{(4v_h k_h - v_l)^2},\end{aligned} \tag{46}$$

Notice that equation 46 is positive because $\left(v_l\frac{dv_h}{dk_h} - v_h\frac{dv_l}{dk_h}\right) > 0$, since $v_l/v_h > \frac{dv_l/dk_h}{dv_h/dk_h}$. Let the marginal consumer indifferent between low-quality certification and the outside uncertified good be $\theta_{0l}(v_h^*(k_h), v_l^*(k_h); k_h)$.

$$\begin{aligned}\frac{d\theta_{0l}}{dk_h} &= \frac{\partial\theta_{0l}}{\partial k_h} + \frac{\partial\theta_{0l}}{\partial v_h}\frac{dv_h}{dk_h} + \frac{\partial\theta_{0l}}{\partial v_l}\frac{dv_l}{dk_h} \\ &= \frac{3v_h v_l}{(4v_h k_h - v_l)^2} + \frac{3k_h v_l}{(4v_h k_h - v_l)^2}\frac{dv_l}{dk_h} - \frac{3k_h v_h}{(4v_h k_h - v_l)^2}\frac{dv_l}{dk_h} \\ &= \frac{3v_h v_l}{(4v_h k_h - v_l)^2} + \left(v_l\frac{dv_h}{dk_h} - v_h\frac{dv_l}{dk_h}\right)\frac{3k_h}{(4v_h k_h - v_l)^2},\end{aligned} \tag{47}$$

which again is positive. One should notice that $\frac{d\theta_{0l}}{dk_h} > \frac{d\theta_{lh}}{dk_h}$. Recall that under the assumption of uniform distribution of tastes the demand for high quality certified products is $D_h = (1 - \theta_{lh})$ and for low quality certified $D_l = (\theta_{lh} - \theta_{0l})$. Therefore when misperception the size of demand decreases for both firms. As a consequence, the total demand for certified products decreases as well. This proves that the quantity of graded products decrease with an increase in the misperception parameter.

Next, we check the effects of increases in k_h in prices. We will show that by checking the effects of misperception on profits, we can readily get the effects of misperception in prices. The effects of misperception on low-quality firm can be stated in equation 48.

$$\begin{aligned}
\frac{\partial \pi_l(v_h^s(k_h), v_l^s(k_h); k_h))}{\partial k_h} &= \frac{\partial R_l}{\partial k_h} + \frac{\partial R_l}{\partial v_h}\frac{dv_h^s}{dk_l} + \frac{\partial R_l}{\partial v_l}\frac{dv_l^s}{dk_h} - \frac{\partial C(v_l)}{\partial v_l}\frac{dv_l^s}{dk_h} \qquad (48)\\
\frac{\partial \pi_l(v_h^s(k_h), v_l^s(k_h); k_h))}{\partial k_h} &= \underbrace{\frac{\partial R_l}{\partial k_h}}_{>0} + \underbrace{\frac{\partial R_l}{\partial v_h}}_{>0}\underbrace{\frac{dv_h^s}{dk_l}}_{>0} + \underbrace{\left(\frac{\partial R_l}{\partial v_l} - \frac{\partial C(v_l)}{\partial v_l}\right)}_{=0}\underbrace{\frac{dv_l^s}{dk_h}}_{>0} > 0\\
&= \frac{v_h v_l^2(2k_h v_h + v_l)}{(4k_h v_h - v_l)^3} + \frac{4k_h^2 v_h^2(2k_h v_h + v_l)}{(4k_h v_h - v_l)^3}\frac{dv_h^s}{dk_h} > 0.
\end{aligned}$$

Thus, increases in k_h raise profits of the low-quality follower. The effects on the leader are stated in equation 49.

$$\begin{aligned}
\frac{\partial \pi_h(v_h^s(k_h), v_l^s(k_h); k_h))}{\partial k_h} &= \frac{\partial R_h}{\partial k_h} + \frac{\partial R_h}{\partial v_h}\frac{dv_h^s}{dk_h} + \frac{\partial R_h}{\partial v_l}\frac{dv_l^s}{dk_h} - \frac{\partial C(v_h)}{\partial v_h}\frac{dv_h}{dk_h} \qquad (49)\\
&= \frac{\partial R_h}{\partial k_h} + \frac{\partial R_h}{\partial v_l}\frac{dv_l^s}{dk_h} + \left(\frac{\partial R_h}{\partial v_h} - \frac{\partial C(v_h)}{\partial v_h}\right)\frac{dv_h^s}{dk_h}\\
&= \frac{\partial R_h}{\partial k_h} + \frac{\partial R_h}{\partial v_l}\frac{dv_l^s}{dk_h} + \left(\frac{\partial R_h}{\partial v_h} - \frac{\partial C(v_h)}{\partial v_h}\right)\frac{dv_h^s}{dk_h}\\
&= \frac{\partial R_h}{\partial k_h} + \frac{\partial R_h}{\partial v_l}\frac{dv_l^s}{dk_h} + \left(\frac{-\partial R_h}{\partial v_l}\frac{dv_l^s}{dv_h}\right)\frac{dv_h^s}{dk_h}\\
&= \frac{\partial R_h}{\partial k_h}\\
&= \frac{4k_h v_h^2(4k_h^2 v_h^2 - 3k_h v_h v_l + 2v_l^2)}{(4k_h v_h - v_l)^3}\\
&> 0 \text{ if } \frac{v_l}{v_h} < \frac{4k_h}{7}
\end{aligned}$$

where we use the fact that $\frac{\partial R_h}{\partial v_h} + \frac{\partial R_h}{\partial v_l}\frac{dv_l}{dv_h} - \frac{\partial C(v_h)}{\partial v_h} = 0$. We conclude that both firm's profit increase when $k_h > 0$.

Finally, we can analyze the effect that an increase in misperception has on prices. We know that firms' profit increase when misperception increases. By $C'(\cdot) > 0$, $dv_h/dk_h > 0$, $dv_l/dk_h > 0$, we know that costs increase when misperception increases. This means that firms' profits increase because revenues must increase more than increases in cost under a higher misperception parameter. But demand decreases as misperception rises. Since revenue is given by the multiplication of prices by firm's demand, we have that prices necessarily must increase under increases in k_h.

Finally, we check how price per unit of quality supplied change when k_h increases. But to check for the ratio price per quality supplied, we need to assess the intensity of price increase vis-a-vis the intensity of quality increase. Notice from equation 44 and 45 that have established the sign of the change in qualities, not their intensity. Since we have not established the intensity of quality change, we solve numerically for changes in price per quality using a quadratic cost. Figure 5 shows the result: we observe an increase in price per quality whenever misperception increase in the direction of overvaluation.

(a) Effect of misperception, k_h

(b) Effect of misperception, k_h

Figure 5: Variation in price/quality with changes in k_h, under sequential quality competition and quadratic costs for quality.

This completes the proof of proposition 1 for sequential quality competition under assumption 1.

Misperception in high-quality, k_h, under simultaneous quality choice.

The simultaneous quality choice case differs only slightly from the sequential quality choice case. Again, we show that 1) quality choices increase 2) that quantity consumed of products decrease 3) that prices increase. One can check that equation 37 holds for the simultaneous case as well. Equation 40 also holds for the simultaneous case, which implies that high quality is a strategic complement to low-quality profits. To show that low quality is a strategic complement to high-quality firms, we must show that $\frac{\partial^2 \pi_h}{\partial v_h \partial v_l} > 0$ in the simultaneous case:

$$\frac{\partial^2 \pi_h}{\partial v_h \partial v_l} = \frac{8k_h^2 v_h v_l (5k_h v_h + v_l)}{(4k_h v_h - v_l)^4} > 0 \tag{50}$$

To prove increasing differences in parameter k_h for both firms, we need $\frac{\partial^2 \pi_l}{\partial v_l \partial k_h} > 0$ and $\frac{\partial^2 \pi_h}{\partial v_h \partial k_h} > 0$. Notice that equation 40 shows that $\frac{\partial^2 \pi_l}{\partial v_l \partial k_h} > 0$. We can show that:

$$\frac{\partial^2 \pi_h}{\partial v_h \partial k_h} = \frac{4k_h v_h (16k_h^3 v_h^3 - 16k_h^2 v_h^2 v_l + k_h v_h v_l^2 - 4v_l^3)}{(4k_h v_h - v_l)^4} > 0, \text{ if } \frac{v_l}{v_h} < \frac{4k_h}{7}, \tag{51}$$

which guarantees strategic complementarity between firms' quality choices and the parameter k_h and proves that qualities are increasing in the misperception parameter.

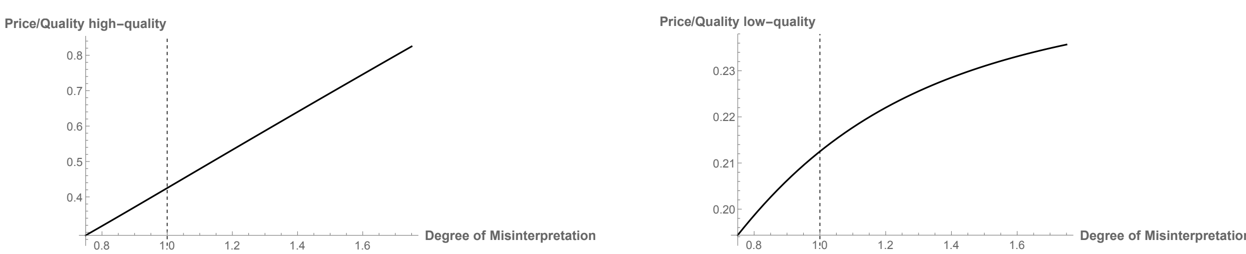
The effects of misperception on the size of the market for high- and low-quality products follow the same structure described in the sequential games (we suppress the demonstration here). This proves that quantity consumed decreases with misperception parameter k_h.

To show the effect of misperception on prices, we show first the effect of misperception on profits first. The effect of misperception parameter on the low-quality firm profits is the same as the one in sequential games – given by equation 48. The profit of the high-quality firm is given by equation 52.

$$\begin{aligned}
\frac{\partial \pi_h(v_h^n(k_h), v_l^n(k_h); k_h))}{\partial k_h} &= \frac{\partial R_h}{\partial k_h} + \frac{\partial R_h}{\partial v_h}\frac{dv_h^n}{dk_h} + \frac{\partial R_h}{\partial v_l}\frac{dv_l^n}{dk_h} - \frac{\partial C(v_h)}{\partial v_h}\frac{dv_h}{dk_h} \\
&= \frac{\partial R_h}{\partial k_h} + \frac{\partial R_h}{\partial v_l}\frac{dv_l^n}{dk_h} + \left(\frac{\partial R_h}{\partial v_h} - \frac{\partial C(v_h)}{\partial v_h}\right)\frac{dv_h^n}{dk_h} \\
&= \frac{\partial R_h}{\partial k_h} + \frac{\partial R_h}{\partial v_l}\frac{dv_l^n}{dk_h} + \underbrace{\left(\frac{\partial R_h}{\partial v_h} - \frac{\partial C(v_h)}{\partial v_h}\right)}_{=0}\frac{dv_h^n}{dk_h} \\
&= \frac{\partial R_h}{\partial k_h} + \frac{\partial R_h}{\partial v_l}\frac{dv_l^n}{dk_h}
\end{aligned} \tag{52}$$

Expression 52 cannot be analytically signed as the second term of the expression is negative and the first term is positive. We numerically solve equation 52 under quadratic quality costs and conclude that profits of the high-quality firm increases. We can use the same rationale of sequential quality competition to conclude that prices increase with increases in k_h under simultaneous quality choice.

Figure 6 uses quadratic costs to numerically solve for variations in price per quality offered by firms when k_h increases.



(a) Effect of misperception, k_h

(b) Effect of misperception, k_l

Figure 6: Variation in price/quality with changes in k_h, under sequential quality competition and quadratic costs for quality.

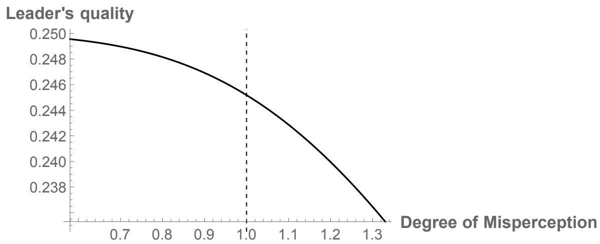
Price per quality increases for high- and low-quality products. This concludes the proof of proposition 1.

Proof of Proposition 2.

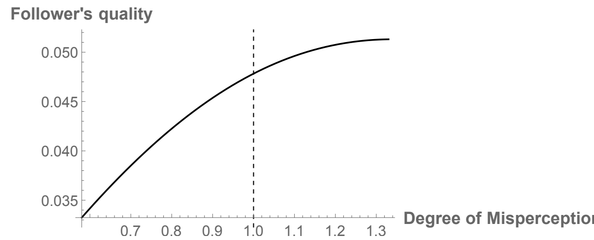
Misperception in high-quality, k_l, under sequential quality choice.

We follow the same procedure as in the comparative statics for misperception in high-quality grades. However, using the same procedures as in Proposition 1, we cannot unequivocally show that there are increasing differences between v_l and k_l, and v_h and k_l. The sign of the comparative statics results for market outcomes and welfare is equally ambiguous and are not shown here. We resort to numerical solutions by imposing quadratic costs and solving the comparative statics for quality, market size, prices, and prices per unit of quality.

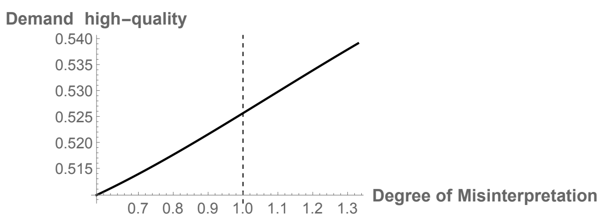
Figure 7 shows the effects of increases in k_l in market outcomes. Panel (a) and panel (b) shows that quality of the high-quality firm decreases with k_l while low-quality increases. Panel (c) and (d) shows that low- and high-quality market share increase with increases in k_l. Figure (e) and (f) shows that high-quality prices decrease with increases in k_l, while low-quality prices increase. Finally, panels (g) and (h) shows the price per quality supplied of high-quality decreases, while price per quality supplied of low-quality products increase.



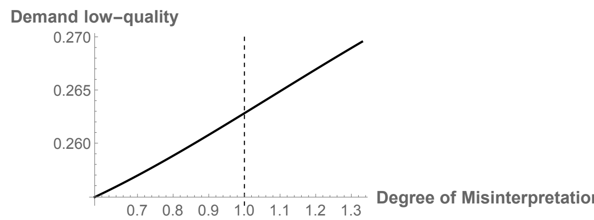
(a) Variation in quality with changes in k_l, under sequential quality competition and quadratic costs for quality.



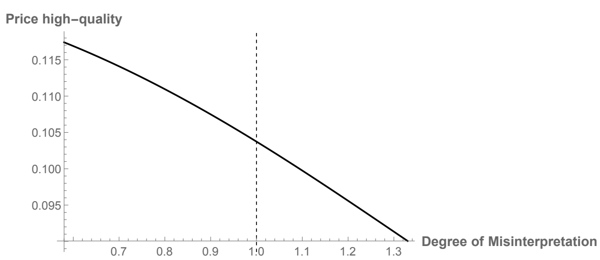
(b) Variation in quality with changes in k_l, under sequential quality competition and quadratic costs for quality.



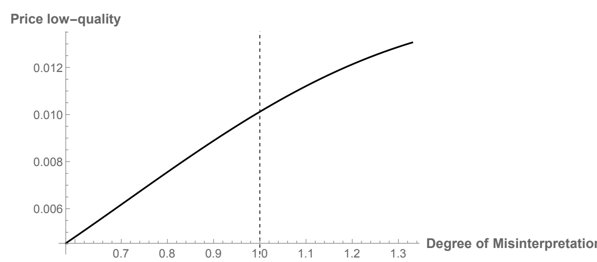
(c) Variation in market size (demand) with changes in k_l, under sequential quality competition and quadratic costs for quality.



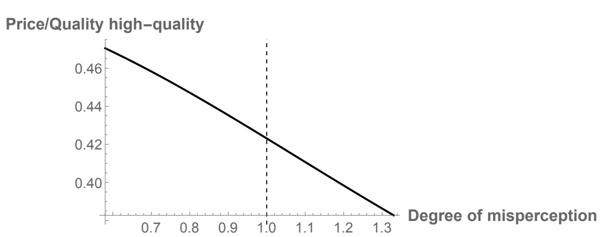
(d) Variation in market size (demand) with changes in k_l, under sequential quality competition and quadratic costs for quality.



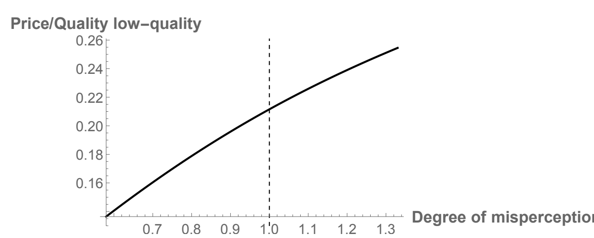
(e) Variation in prices with changes in k_l, under sequential quality competition and quadratic costs for quality.



(f) Variation in prices with changes in k_l, under sequential quality competition and quadratic costs for quality.



(g) Variation in price/quality with changes in k_l, under sequential quality competition and quadratic costs for quality.

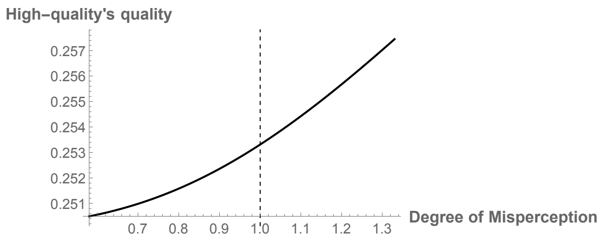


(h) Variation in price/quality with changes in k_l, under sequential quality competition and quadratic costs for quality.

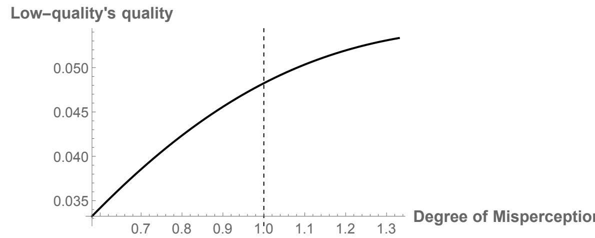
Figure 7: Effects of increases k_l on market outcomes under sequential quality competition

Misperception in low-quality, k_l, under simultaneous quality choice.

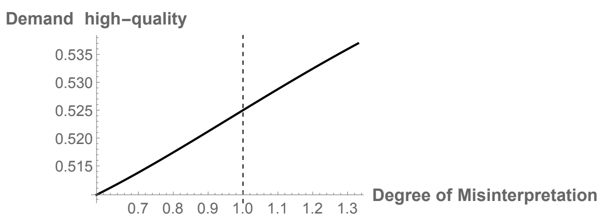
Under the impossibility to unequivocally algebraically sign the effects of k_l on market outcomes under simultaneous quality choices, we use numerical solutions under quadratic costs. These effects are represented in Figure 8.



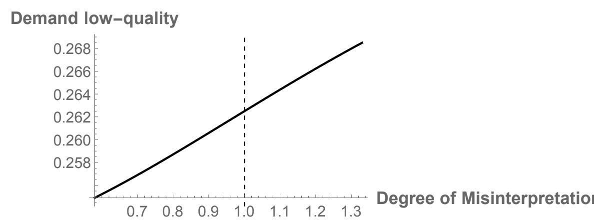
(a) Variation in quality with changes in k_l, under simultaneous quality competition and quadratic costs for quality.



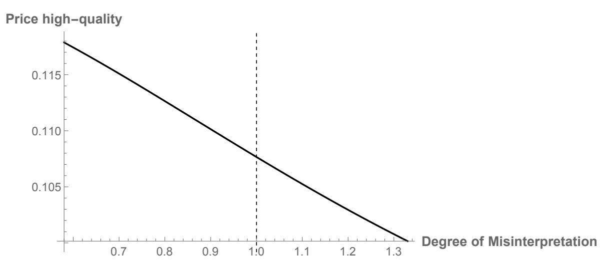
(b) Variation in quality with changes in k_l, under simultaneous quality competition and quadratic costs for quality.



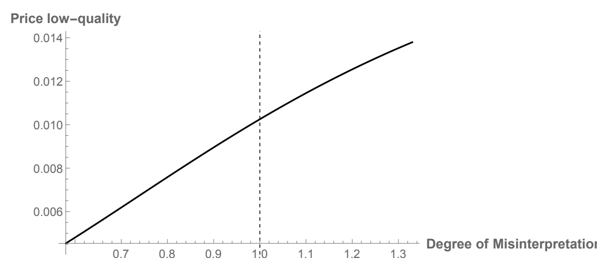
(c) Variation in market size (demand) with changes in k_l, under simultaneous quality competition and quadratic costs for quality.



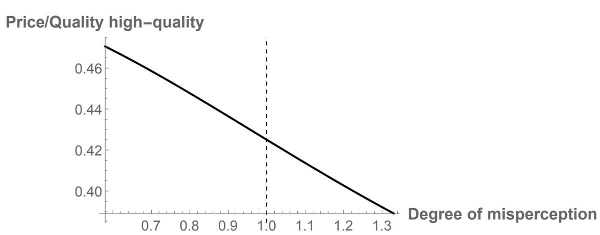
(d) Variation in market size (demand) with changes in k_l, under simultaneous quality competition and quadratic costs for quality.



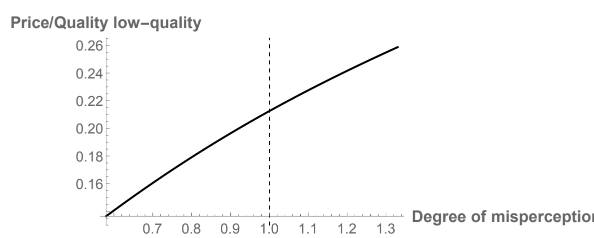
(e) Variation in prices with changes in k_l, under simultaneous quality competition and quadratic costs for quality.



(f) Variation in prices with changes in k_l, under simultaneous quality competition and quadratic costs for quality.



(g) Variation in price/quality with changes in k_l, under simultaneous quality competition and quadratic costs for quality.



(h) Variation in price/quality with changes in k_l, under simultaneous quality competition and quadratic costs for quality.

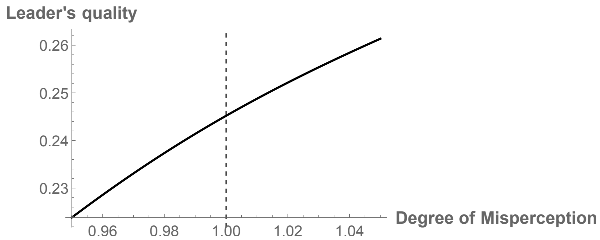
Figure 8: Effects of increases k_l on market outcomes under simultaneous quality competition

According to Figure 8, high- and low-quality levels increase with k_l; prices of the high-quality decreases with k_l, while low-quality prices increase; the demand for both products increase; and the price per unit of quality supplied increase for low-quality, but decreases for high-quality. This proves Proposition 2.

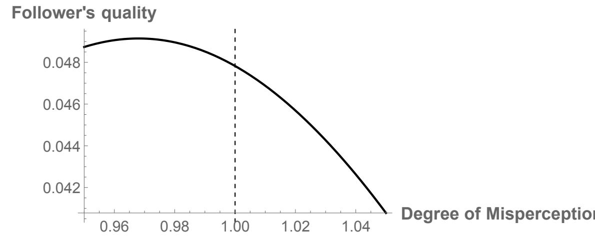
Proof of Proposition 3

Mean-preserving misperception under sequential and simultaneous quality misperception.

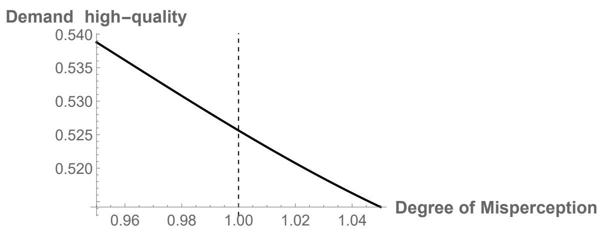
We provide full numerical results under quadratic costs. The numerical results are limited to the interval in which we checked that the no leapfrogging condition holds, i.e. to $k_h \in (0.95,\ 1.05)$. We provide results for sequential quality competition (Figure 9) and simultaneous quality competition (Figure 10).



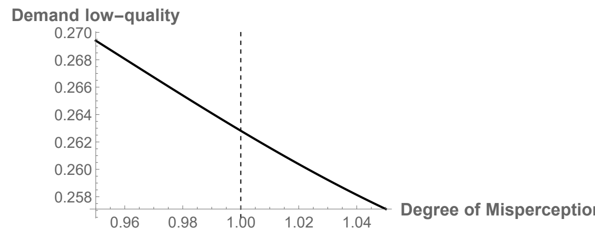
(a) Variation in quality with changes in $k_h(k_l)$, under sequential quality competition and quadratic costs for quality.



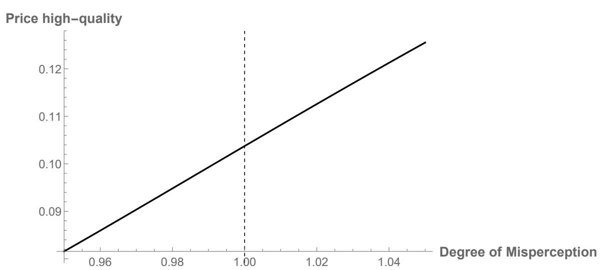
(b) Variation in quality with changes in $k_h(k_l)$, under sequential quality competition and quadratic costs for quality.



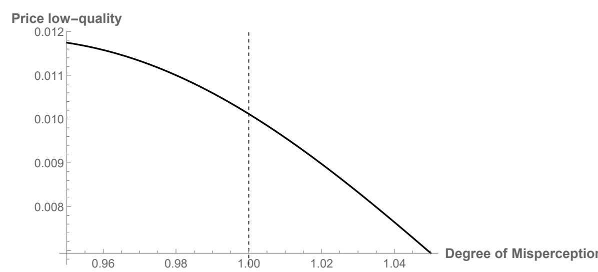
(c) Variation in market size (demand) with changes in $k_h(k_l)$, under sequential quality competition and quadratic costs for quality.



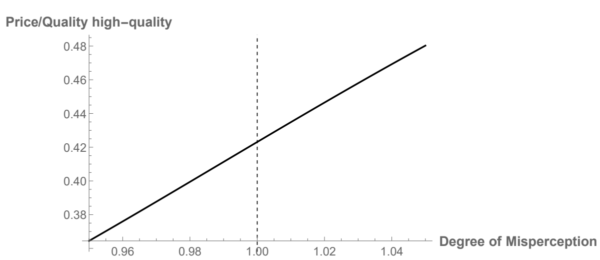
(d) Variation in market size (demand) with changes in $k_h(k_l)$, under sequential quality competition and quadratic costs for quality.



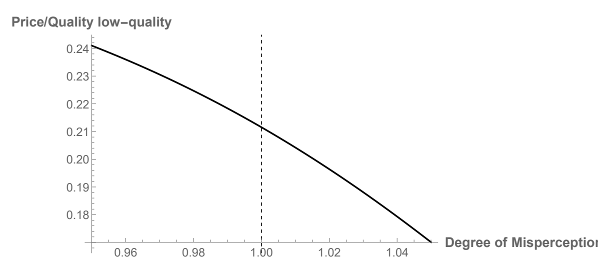
(e) Variation in prices with changes in $k_h(k_l)$, under sequential quality competition and quadratic costs for quality.



(f) Variation in prices with changes in $k_h(k_l)$, under sequential quality competition and quadratic costs for quality.

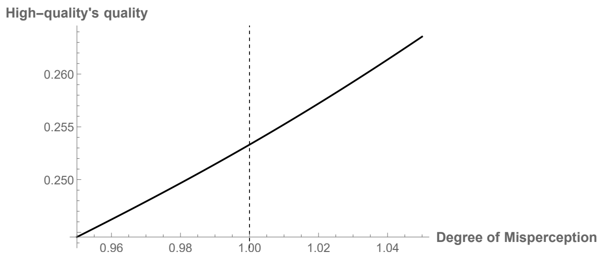


(g) Variation in price/quality with changes in $k_h(k_l)$, under sequential quality competition and quadratic costs for quality.

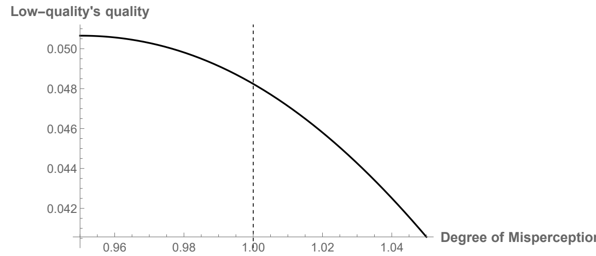


(h) Variation in price/quality with changes in $k_h(k_l)$, under sequential quality competition and quadratic costs for quality.

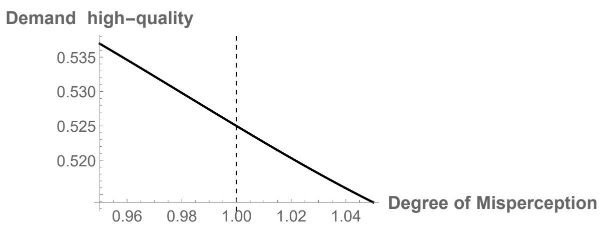
Figure 9: Effects of increases k_h on market outcomes under sequential quality competition and mean-preserving misperception



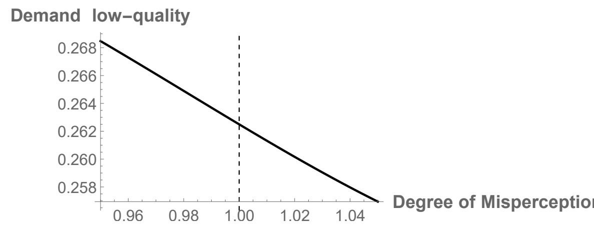
(a) Variation in quality with changes in $k_h(k_l)$, under simultaneous quality competition and quadratic costs for quality.



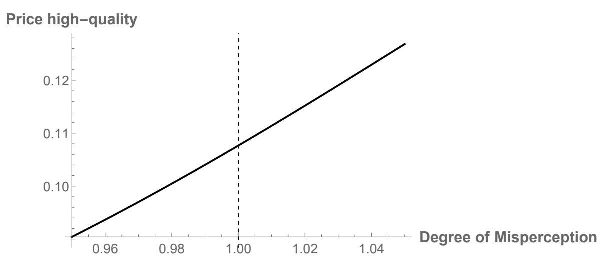
(b) Variation in quality with changes in $k_h(k_l)$, under simultaneous quality competition and quadratic costs for quality.



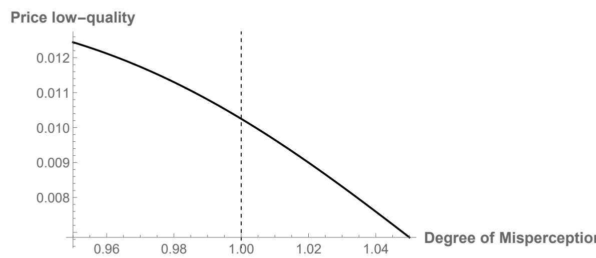
(c) Variation in market size (demand) with changes in $k_h(k_l)$, under simultaneous quality competition and quadratic costs for quality.



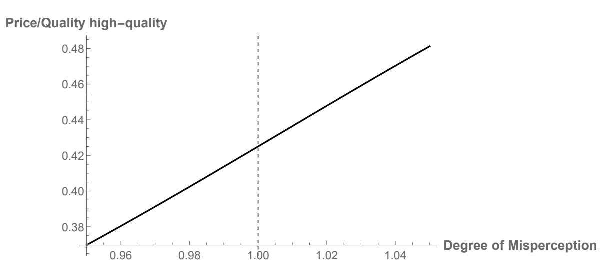
(d) Variation in market size (demand) with changes in $k_h(k_l)$, under simultaneous quality competition and quadratic costs for quality.



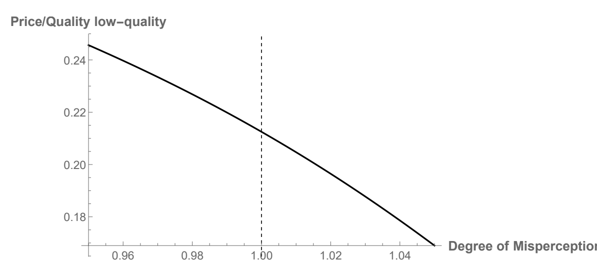
(e) Variation in prices with changes in $k_h(k_l)$, under simultaneous quality competition and quadratic costs for quality.



(f) Variation in prices with changes in $k_h(k_l)$, under simultaneous quality competition and quadratic costs for quality.



(g) Variation in price/quality with changes in $k_h(k_l)$, under simultaneous quality competition and quadratic costs for quality.



(h) Variation in price/quality with changes in $k_h(k_l)$, under simultaneous quality competition and quadratic costs for quality.

Figure 10: Effects of increases k_h on market outcomes under simultaneous quality competition and mean-preserving misperception

Proof of Claim 1.

Misperception in high-quality grades, k_h

We start with the case of misperception in high-quality products. We show the conditions for which firms underprovide quality relative to a social planner that wants to maximize welfare. Let the welfare function, $W(\cdot)$ be the sum of consumer surplus and total profit of firms, such that $W(v_h, v_l; k_h) = CS_h(v_h, v_l; k_h) + CS_l(v_h, v_l; k_h) + Rh(v_h, v_l; k_h) - C(v_h) + Rh(v_h, v_l; k_h) - C(v_l)$.

We first show the conditions for which the low-quality firm underprovide quality. We follow Buehler and Schuett (2014) and let $NW(v_h, v_l; k_h) = W(v_h, v_l; k_h) + C(v_h) + C(v_l)$. We can differentiate both sides with respect to v_l and obtain $\frac{\partial NW}{\partial v_l} = \frac{\partial W}{\partial v_l} + \frac{\partial C(v_l)}{\partial v_l}$, which we can rearrange to $\frac{\partial W}{\partial v_l} = \frac{\partial NW}{\partial v_l} - \frac{\partial C(v_l)}{\partial v_l}$. From Appendix 1, we know that the optimal quality of the simultaneous quality game, v_l^n is larger than the sequential quality game v_l^s. Thus, to show that there exist underprovision of quality in our setting, it is sufficient to show that, at the solution of the simultaneous quality game, $\{v_h^n,\ v_l^n\}$, $\frac{\partial W}{\partial v_l}\ |_{v_i=v_i^n} = \frac{\partial NW}{\partial v_l} - \frac{\partial C}{\partial v_l}\ |_{v_i=v_i^n} > \frac{\partial \pi_l}{\partial v_l}|_{v_i=v_i^n} = \frac{\partial R_l}{\partial v_l} - \frac{\partial C}{\partial v_l}\ |_{v_i=v_i^n}$. Using the definitions above, we need to show inequality 53 holds.

$$
\begin{aligned}
&\frac{\partial NW}{\partial v_l} - \frac{\partial C(v_l)}{\partial v_l} > \frac{\partial R_l}{\partial v_l} - \frac{\partial C(v_l)}{\partial v_l} \\
&\frac{\partial NW}{\partial v_l} > \frac{\partial R_l}{\partial v_l} \\
&\frac{\partial}{\partial v_l}\left(\int_{\theta_{lh}}^{1} \theta v_h d\theta + \int_{\theta_{0l}}^{\theta_{lh}} \theta v_l d\theta\right) > \frac{\partial R_l}{\partial v_l} \\
&\frac{k_h v_h^2(4k_h(3k_h+2) - (13k_h+4)v_l)}{2(4k_h v_h - v_l)^3} > \frac{k_h^2 v_h^2(4_{kh} v_h - 7v_l)}{(4k_h v_h - v_l)^3}
\end{aligned}
\tag{53}
$$

which are satisfied by $0.75 < k_h < 1.75$.

Now, we want to show the conditions for which the low-quality firm underprovide quality. Again, it is sufficient to show that at the solution of the simultaneous quality game, $\{v_h^n, v_l^n\}$, $\frac{\partial W}{\partial v_h}\ |_{v_i=v_i^n} = \frac{\partial NW}{\partial v_h} - \frac{\partial C}{\partial v_h}\ |_{v_i=v_i^n} > \frac{\partial \pi_h}{\partial v_h}|_{v_i=v_i^n} = \frac{\partial R_h}{\partial v_h} - \frac{\partial C}{\partial v_l}\ |_{v_i=v_i^n}$. Using the definitions above, we need to show inequality 54 holds.

$$\begin{aligned}
&\frac{\partial NW}{\partial v_l} - \frac{\partial C(v_l)}{\partial v_l} > \frac{\partial R_l}{\partial v_l} - \frac{\partial C(v_l)}{\partial v_l} \\
&\frac{\partial NW}{\partial v_l} > \frac{\partial R_l}{\partial v_l} \\
&\frac{\partial}{\partial v_l}\left(\int_{\theta_H}^{1} \theta v_h d\theta + \int_{\theta_l}^{\theta_h} \theta v_l d\theta\right) > \frac{\partial R_l}{\partial v_l} \\
&\frac{k_h v_h^2 (4k_h(3k_h+2) - (13k_h+4)v_l)}{2(4k_h v_h - v_l)^3} > \frac{k_h^2 v_h^2 (4_{kh} v_h - 7v_l)}{(4k_h v_h - v_l)^3},
\end{aligned} \tag{54}$$

which are satisfied by $0.75 < k_h < 1.33$.

Misperception in low-quality grades, k_l.

We use the same logic to show the range in which firms underprovide quality under misperception in the low-quality label. Again, notice that to show underprovision of quality, it is sufficient to evaluate whether the gradients of the welfare function are greater than the gradient of the profit function at simultaneous quality competition; formally, we want to show that $\frac{\partial W}{\partial v_i}\,|_{v_i=v_i^n} = \frac{\partial NW}{\partial v_i} - \frac{\partial C}{\partial v_i}\,|_{v_i=v_i^n} > \frac{\partial \pi_i}{\partial v_i}|_{v_i=v_i^n} = \frac{\partial R_i}{\partial v_i} - \frac{\partial C}{\partial v_i}|_{v_i=v_i^n}$ under a given k_i, where $i \in \{h, l\}$. Equations 55 and 56 show the conditions:

$$\begin{aligned}
&\frac{\partial NW}{\partial v_l} - \frac{\partial C}{\partial v_l}\bigg|_{v_i=v_i^n} > \frac{\partial R_l}{\partial v_l} - \frac{\partial C}{\partial v_l}\bigg|_{v_i=v_i^n} \\
&\frac{\partial NW}{\partial v_l}\bigg|_{v_i=v_i^n} > \frac{\partial R_l}{\partial v_l}\bigg|_{v_i=v_i^n} \\
&\frac{v_h^2\left(4\left(3+2k_l\right)v_h - rk_l\left(13+4k_l\right)v_h\right)}{2\left(4v_h - rk_l v_h\right)^3} > \frac{k_l v_h^2\left(4v_h - 7rk_l v_h\right)}{4v_h - ak_l v_h}
\end{aligned} \tag{55}$$

such that the inequality is satisfied under $0.58 < k_l < 1.33$ and under quadratic costs. For high-quality:

$$\frac{\partial NW}{\partial v_h} - \frac{\partial C}{\partial v_h}\bigg|_{v_i=v_i^n} > \frac{\partial R_h}{\partial v_h} - \frac{\partial C}{\partial v_h}\bigg|_{v_i=v_i^n} \tag{56}$$
$$\frac{\partial NW}{\partial v_h}\bigg|_{v_i=v_i^n} > \frac{\partial R_h}{\partial v_h}\bigg|_{v_i=v_i^n}$$
$$\frac{24v_h^3 - 18rk_l v_h^3 + r^3k_l^2v_h^3 + r^2k_l\left(1+4k_l\right)v_h^3}{\left(4v_h - rk_lv_h\right)^3} > \frac{k_lv_h^2\left(4v_h - 7rk_lv_h\right)}{4v_h - ak_lv_h}$$

such that the inequality is satisfied under $0.58 < k_l < 1.33$ and under quadratic costs. This proves Claim 1.

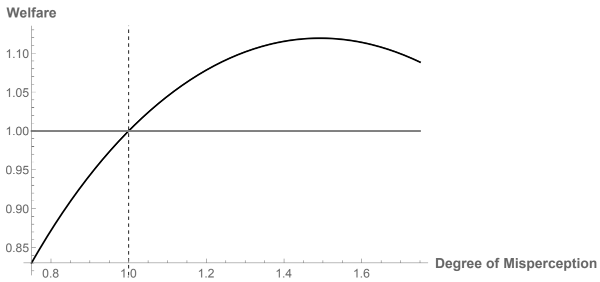
Proof of Proposition 4

We show that overestimation of high-quality grades relative to low-quality grades can increase total welfare. We established in the text that the welfare function is given by total valuation of grades minus the cost to supply them. Thus, welfare evaluated at the subgame perfect Nash equilibrium quality level (either for simultaneous or sequential quality competition) is given by $W(v_h^*(k_i), v_l^*(k_i); k_i) = TV(v_h^*(k_i), v_l^*(k_h); k_i) - C(v_h^*) - C(v_i^*)$, $i \in \{h, l\}$ where TV represents total value. The effect of increasing k_i in welfare is given by equation 57.

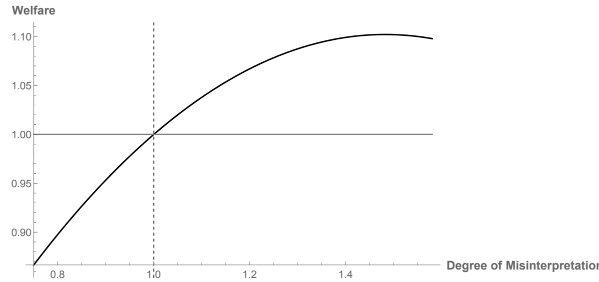
$$\begin{aligned}\frac{dW}{dk_i} &= \frac{dTV}{dk_i} + \left(\frac{\partial TV}{\partial v_h} - \frac{\partial C}{\partial v_h}\right)\frac{dv_h^*}{dk_i} + \left(\frac{\partial TV}{\partial v_l} - \frac{\partial C}{\partial v_l}\right)\frac{dv_l^*}{dk_i} \\ &= -\left(\theta_{lh}v_h\right)\frac{d\theta_{lh}}{dk_i} + \left(\theta_{lh}v_l\right)\frac{d\theta_{lh}}{dk_i} - \left(\theta_{0l}v_l\right)\frac{d\theta_{0l}}{dk_i} + \left(\frac{\partial TV}{\partial v_h} - \frac{\partial C}{\partial v_h}\right)\frac{dv_h^*}{dk_i} + \left(\frac{\partial TV}{\partial v_l} - \frac{\partial C}{\partial v_l}\right)\frac{dv_l^*}{dk_i}\end{aligned} \tag{57}$$

First, notice that $\frac{dv_h^*}{dk_h} > 0$ and $\frac{dv_l^*}{dk_h} > 0$ in sequential and simultaneous quality competition. The first term in equation 57 shows how the change in market size due to increasing k_h changes welfare. Notice that dTV/dk_h is decomposed in the first 3 terms. These terms only change market size because they only affect the marginal consumers, as we are holding qualities at their optimal equilibrium under quality competition. When misperpcetion is in the high-quality grade, we can show that these 3 first terms are negative under sequential and simultaneous quality competition, as total market size decreases (as shown in Proposition 1 and 2). In relation to the social optimum equilibrium, we showed in Claim 1 that the second term and third terms of equation 57 are positive when evaluated at the sub-game perfect Nash equilibrium, for at least a given level of $k_h = 1$. Notice these terms would be zero if evaluated under the social planner problem. From Claim 1, we can show numerically in Figure 11 that under assumption 1 and assumption 3, welfare increases up to a given k_h, such that $k_h > 1$.

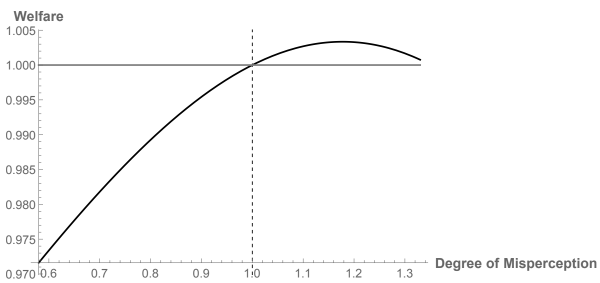
For the case in misperpcetion of low-quality grades, notice that from Proposition 2, we know that the first 3 terms are positive (market size effect). In simultaneous quality competition, for quadratic costs, terms 4 and 5 are also positive, while term 4 is negative and term 5 is positive under sequential quality competition. We numerically solve for the opposed signs of the comparative statics to show sign the comparative statics, as seen in Figure 11.



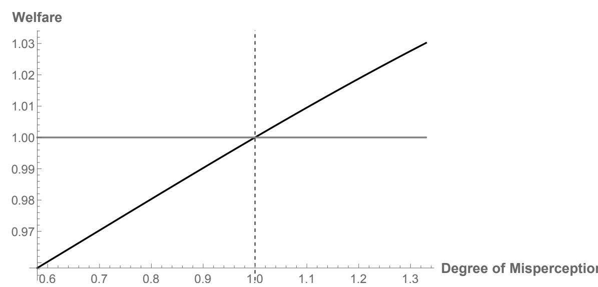
(a) Variation in welfare with changes in k_h, under sequential quality competition



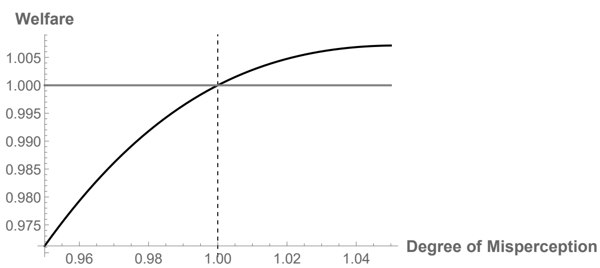
(b) Variation in welfare with changes in k_h, under simultaneous quality competition



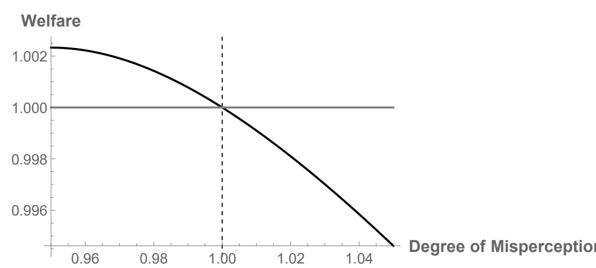
(c) Variation in welfare with changes in k_l, under sequential quality competition



(d) Variation in welfare with changes in k_l, under simultaneous quality competition



(e) Variation in welfare with changes in $k_h(k_l)$, under sequential quality competition and quadratic costs



(f) Variation in welfare with changes in $k_h(k_l)$, under simultaneous quality competition and quadratic costs

Figure 11: Effects of increases of misperception in efficiency (total welfare), normalized to perfect information case.

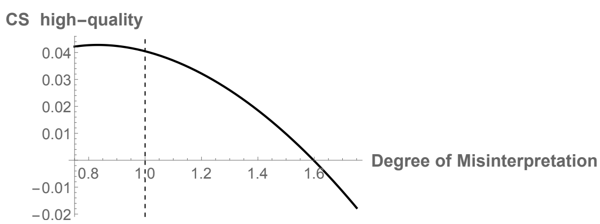
Proof of Proposition 6, 7 and 8

We showed that profits increase with overestimation as part of the proof in proposition 1. In proposition 2, we showed that profits of low-quality firms increase, while high-quality firm's profit decreases in both simultaneous and sequential quality competition. We show the effects of overestimation in consumer surplus next. Equations 58 and 59 decompose the effects of changes in misperception in consumer surplus for high-quality consumer and low-quality consumers, respectively.

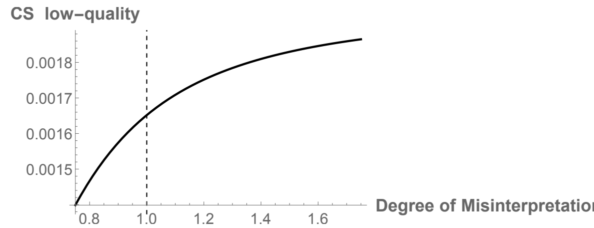
$$\frac{d}{dk_i}\int_{\theta_{lh}}^{1}\theta v_h - p_h d\theta = (v_h - p_h)\frac{d1}{dk_i} - (\theta_{lh}v_h - p_h)\frac{d\theta_{lh}}{dk_i} + \int_{\theta_{lh}}^{1}\frac{\partial}{\partial k_i}\theta v_h - p_h \tag{58}$$
$$= -(\theta_{lh}v_h - p_h)\frac{d\theta_{lh}}{dk_h} + \frac{1-\theta_{lh}^2}{2}\frac{dv_h}{dk_h} - (1-\theta_{lh})\frac{dp_h}{dk_h}$$

$$\frac{d}{dk_i}\int_{\theta_{0l}}^{\theta_{lh}}\theta v_l - p_l d\theta = (\theta_{lh}v_l - p_l)\frac{d\theta_{lh}}{dk_i} - (\theta_{0l}v_l - p_l)\frac{d\theta_{0l}}{dk_i} + \int_{\theta_{lh}}^{\theta_{lh}}\frac{\partial}{\partial k_i}\theta v_l - p_l \tag{59}$$
$$= (\theta_{lh}v_l - p_l)\frac{d\theta_{lh}}{dk_h} - (\theta_{0l}v_l - p_l)\frac{d\theta_{0l}}{dk_h} + \frac{\theta_{lh}^2 - \theta_{0l}^2}{2}\frac{dv_l}{dk_h} - (\theta_{lh} - \theta_{0l})\frac{dp_l}{dk_h}$$

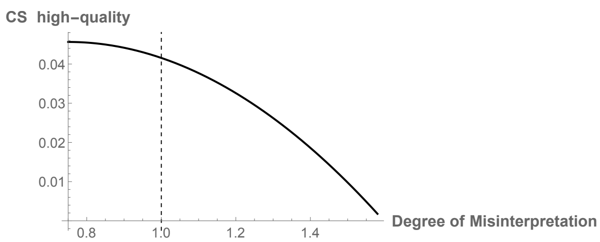
where $i \in \{h, l\}$. From Propositions 1 and 2, we can check that equations 58 and 59 cannot be signed unequivocally. We resort to a numerical solution under quadratic costs to sign the comparative statics. The numerical solution in Figure 12 reveals that consumer surplus for high (low)-quality consumers decrease under misperception of high(low)-quality. Consumer surplus increases for the consumers that do not face misperception of quality.



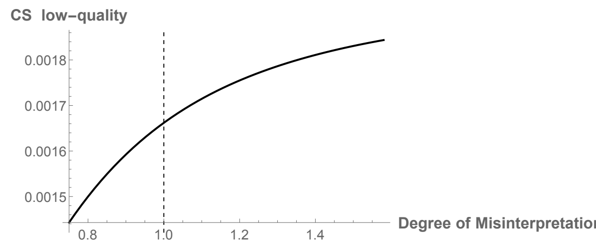
(a) Variation in CS_h with changes in k_h, under sequential quality competition



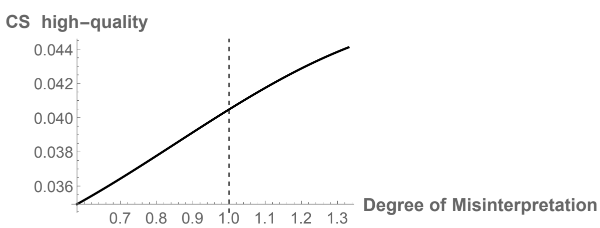
(b) Variation in CS_l with changes in k_h, under sequential quality competition



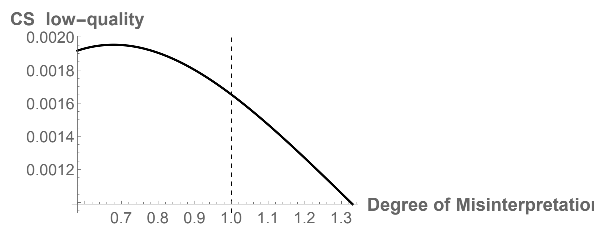
(c) Variation in CS_h with changes in k_h, under simultaneous quality competition



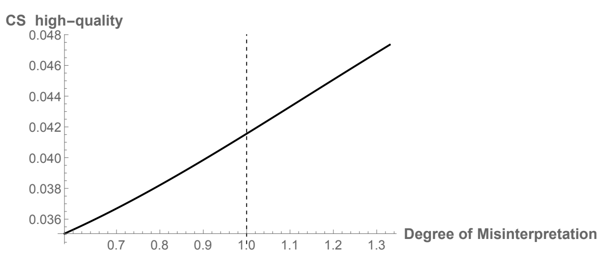
(d) Variation in CS_l with changes in k_h, under simultaneous quality competition



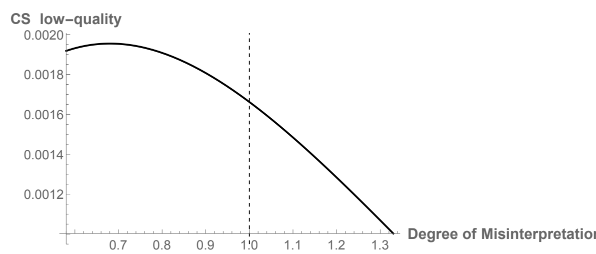
(e) Variation in CS_h with changes in k_l, under sequential quality competition



(f) Variation in CS_l with changes in k_l, under sequential quality competition



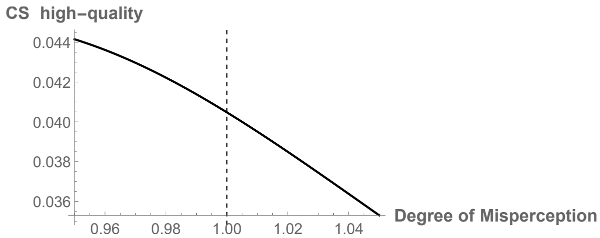
(g) Variation in CS_h with changes in k_l, under simultaneous quality competition



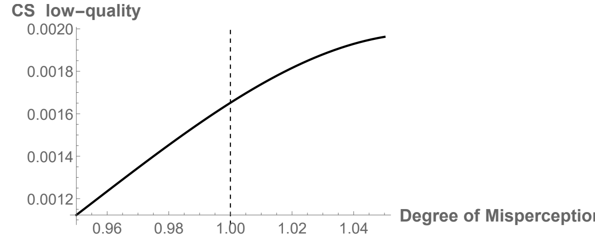
(h) Variation in CS_l with changes in k_l, under simultaneous quality competition

Figure 12: Effects of increases misperception on Consumer Surplus

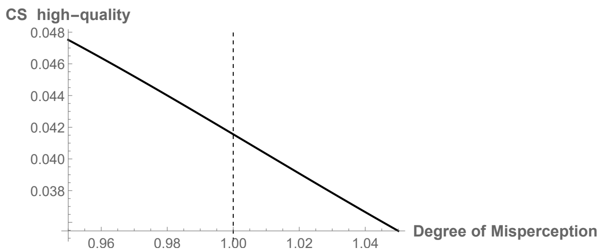
We remain to analyze distribution of welfare under mean-preserving misperception, (k_h and k_l). We can decompose the effects of misperception between the partial effects of k_h and the effects of k_l on $C.S._h$, $C.S._l$, π_h, and π_l. The numerical analysis is shown on Figure 13.



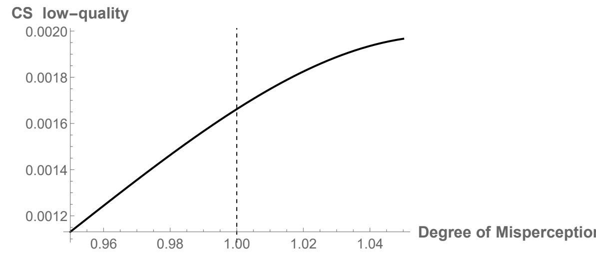
(a) Variation in CS_h with changes in $k_h(k_l)$, under sequential quality competition



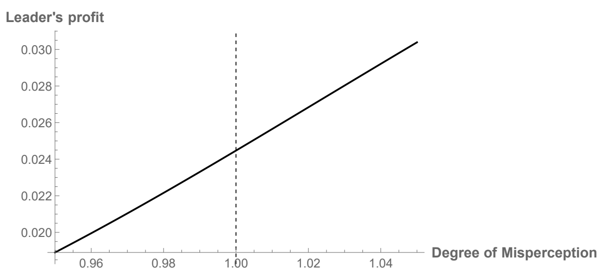
(b) Variation in CS_l with changes in $k_h(k_l)$, under sequential quality competition



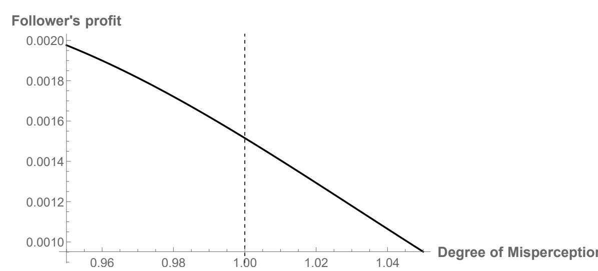
(c) Variation in CS_h with changes in $k_h(k_l)$, under simultaneous quality competition



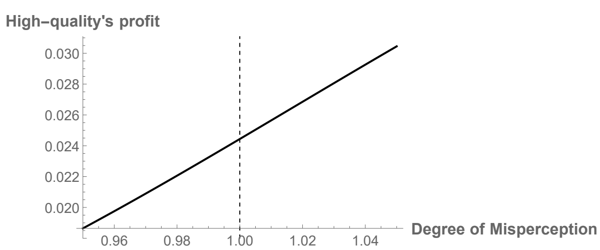
(d) Variation in CS_l (demand) with changes in $k_h(k_l)$, under simultaneous quality competition



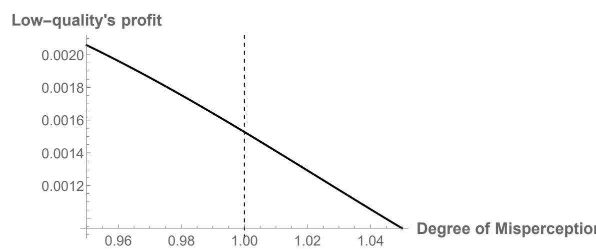
(e) Variation in π_h with changes in $k_h(k_l)$, under sequential quality competition



(f) Variation in π_l with changes in $k_h(k_l)$, under sequential quality competition



(g) Variation in π_h with changes in $k_h(k_l)$, under simultaneous quality competition



(h) Variation in π_l with changes in $k_h(k_l)$, under simultaneous quality competition

Figure 13: Effects of mean-preserving misperception on distribution

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